Chapter 3
Design of Reinforced Concrete Slabs

3.0 NOTATION

a'	Compression face to point on surface of concrete where crack width is calculated
a_b	Centre-to-centre distance between bars or groups of bars
a_{cr}	Point on surface of concrete to nearest face of a bar
A_c	Gross area of concrete in a section
A_s	Area of steel in tension
A_s'	Area of steel in compression
A_{sb}	Minimum area of reinforcement at bottom of slab
A_{st}	Minimum area of reinforcement at top of slab
A_{sx}	Reinforcement in y-direction to resist M_x about x-axis
A_{sy}	Reinforcement in x-direction to resist M_y about Y-axis
A_{sbx}	Area of inclined shear reinforcement to resist V_x
A_{sby}	Area of inclined shear reinforcement to resist V_y
A_{sv}	Area of vertical shear reinforcement
A_{svx}	Area of vertical shear reinforcement to resist V_x
A_{svy}	Area of vertical shear reinforcement to resist V_y
b	Width of reinforced concrete section
b_t	Width of section at centroid of tensile steel
c_{min}	Minimum cover to tensile reinforcement
d	Effective depth of tensile reinforcement
d'	Effective depth of compressive reinforcement
d_l	Distance from tension face of concrete section to centre of tensile reinforcement
E_c	Modulus of elasticity of concrete
E_s	Modulus of elasticity of steel
f_y	Characteristic yield strength of steel
f_y'	Revised compressive stress in steel taking into account depth of neutral axis
f_{cu}	Characteristic cube strength of concrete at 28 days
f_{yv}	Characteristic yield strength of reinforcement used as links
F	Coefficient for calculation of cracked section moment of inertia
G	Shear modulus
h	Overall depth of slab
H	Shorter dimension of a rectangular panel of slab for use of yield-line charts
I	Moment of inertia using b as unit width for slab
l	Clear span or span face-to-face of support

l_e	Effective span
l_o	Centre-to-centre distance between supports
L	Longer dimension of a rectangular panel of slab for yield line calculations
m	Modular ratio $= E_s/E_c$
M_d	Design bending moment per unit width of slab modified to account for axial load
M_x	Moment per unit width about x-axis
M_y	Moment per unit width about y-axis
M_{xy}	Torsional moment per unit width
M_{xt}	Wood–Armer design moment for top reinforcement in y-direction
M_{xb}	Wood–Armer design moment for bottom reinforcement in y-direction
M_{yt}	Wood–Armer design moment for top reinforcement in x-direction
M_yb	Wood–Armer design moment for bottom reinforcement in x-direction
M_{HN}	Ultimate negative moment capacity of slab per unit width about an axis parallel to H
M_{HP}	Ultimate positive moment capacity of slab per unit width about an axis parallel to H
M_{VN}	Ultimate negative moment capacity of slab per unit width about an axis parallel to L
M_{VP}	Ultimate positive moment capacity of slab per unit width about an axis parallel to L
N_x	Axial load per unit width of slab in x-direction to be combined with M_y
N_y	Axial load per unit width of slab in y-direction to be combined with M_x
p	Percentage of tensile reinforcement
p'	Percentage of compressive reinforcement
p_x	Percentage of tensile steel to resist M_x about x-axis
p_y	Percentage of tensile steel to resist M_y about y-axis
r	Loading per unit area used in yield-line analysis (kN/m^2)
r_u	Ultimate loading per unit area
R	Restraint factor for computation of early thermal cracking
R_u	Ultimate total load on panel of slab
S_v	Spacing of vertical links
S_{bx}	Spacing of inclined shear reinforcement to resist V_x per unit width
S_{by}	Spacing of inclined shear reinforcement to resist V_y per unit width
S_{vx}	Spacing of vertical shear reinforcement to resist V_x per unit width
S_{vy}	Spacing of vertical shear reinforcement to resist V_y per unit width
T_1	Differential temperature in a concrete pour for calculation of early thermal cracking
U_n	Perimeter of concentrated load on slab at prescribed multiples of effective depth
U_o	Perimeter of concentrated load footprint on slab
v_c	Design concrete shear strength
v_n	Calculated punching shear stress at perimeter U_n
v_x	Calculated shear stress in concrete due to V_x
v_y	Calculated shear stress in concrete due to V_y
v_{cx}	Design concrete shear stress to compare with V_x for bending about x-axis
v_{cy}	Design concrete shear stress to compare with V_y for bending about y-axis

v_1	Calculated punching shear stress at perimeter U_1
V_x	Shear force per unit width for bending about x-axis
V_y	Shear force per unit width for bending about y-axis
W_{max}	Maximum crack width (mm)
x	Depth of neutral axis from compression face
x	Distance from edge in L-direction to start of a yield line
y	Distance from edge in H direction to start of a yield line
z	Depth of lever arm
α	Angle of inclination to horizontal of shear reinforcement
α	Coefficient of thermal expansion of concrete
β	Angle of inclination to horizontal of concrete strut in truss analogy
β_b	Ratio of redistributed moment over elastic analysis moment
γ_m	Material factor
ε_h	Calculated strain in concrete at depth h
ε_m	Strain with stiffening effect corrected
ε_r	Tensile strain in concrete due to temperature differential causing early thermal cracking
ε_s	Strain at centre of steel reinforcement
ε_s'	Strain at centre of compressive reinforcement
ε_{mh}	Strain at depth h corrected for stiffening effect
ε_l	Calculated strain in concrete ignoring stiffening effect
ρ_{crit}	Critical percentage of steel required to distribute early thermal cracking

3.1 ANALYSIS OF SLABS

3.1.1 Slabs: properties

3.1.1.1 Effective spans

Simply supported or encastré l_e = smaller of $(l + d)$ or l_o

Continuous $l_e = l_o$

Cantilever $l_e = l + \dfrac{d}{2}$

where l_o = centre-to-centre distance between supports
l_e = effective span
l = clear span or span to face of support
d = effective depth of tension reinforcement.

3.1.1.2 Moment of inertia

Method 1 Gross concrete section only
See Section 2.1.3 – use Table 11.2 with b equal to unity.

Method 2 Uncracked transformed concrete
See Section 2.1.3 – use Table 11.2 with b equal to unity and A_s and A_s'

are for unit width. Convert A_s and A_s' into equivalent concrete areas by multiplying by $m = E_s/E_c$. Moment of inertia increment due to steel $= mA_s(x')^2$ where x' is the distance of the steel from the centroidal axis of the section. The shift of the centroidal axis due to the presence of reinforcing steel may be neglected.

Method 3 Average of gross concrete section and cracked section

$$I = 0.5\left(\frac{1}{12}bh^3 + Fbh^3\right)$$

where I = moment of inertia of rectangular concrete section
b = unit width of slab
h = overall depth of slab
F = factor – see Fig. 11.1 for values of F

$$p = \frac{100A_s}{bd}$$

where A_s = area of tensile reinforcement per unit width of slab

$$p' = \frac{100A_s'}{bd}$$

where A_s' = area of compressive reinforcement per unit width of slab

$$m = \text{modular ratio} = \frac{E_s}{E_c}$$

Note: For slabs, b is taken equal to unity.
The preferred method is Method 3 for rectangular sections. Where reinforcement quantities are not known, an assumption may be made of the percentage of reinforcement.

3.1.1.3 Modulus of elasticity

See Section 2.1.4.

3.1.1.4 Shear modulus

Shear modulus $G = 0.42E_c$ for concrete.

3.1.1.5 Poisson's ratio

Poisson's ratio for concrete is 0.2

3.1.1.6 Thermal strain

See Section 2.1.9.

3.1.2 Analysis of slabs

The objective is to find the following internal forces by analysis:

(1) Moments M_x, M_y and M_{xy}

(2) Shears V_x and V_y
(3) Wood–Armer moments M_{xt}, M_{xb}, M_{yt} and M_{yb}
(4) In-plane loads N_x and N_y

Method 1
BS 8110: Part 1: 1985, clauses 3.5.2 and 3.5.3, Table 3.15.[1]

Method 2
Yield-line method: non-linear – use Figs 3.18 to 3.33.

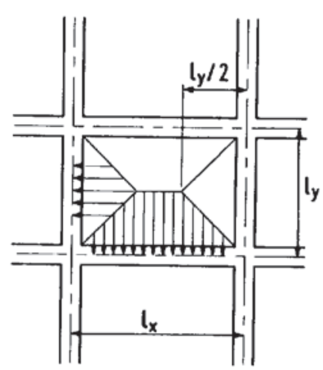
Method 3
Finite difference: linear elastic – Moody's table.[9]

Method 4
Finite element analysis: linear elastic – use general purpose computer program or Figs 3.1 to 3.17.

Commentary
Method 1 is a non-conservative approach. If cracking has to be avoided, an elastic method of analysis, i.e. finite element or finite difference, will be more appropriate. For complicated loadings and complex layout of slab panels and supporting arrangements, it is always recommended to use finite element analysis. Finite element analysis will give Wood-Armer design moments for top and bottom reinforcement in a panel of slab. Method 2 (yield-lines) may be successfully used for uniformly loaded slab panels with different boundary conditions. Method 2 gives a better representation of internal forces in a slab panel than Method 1.

Recommendations
Use Method 2 or Method 3 generally. Use Method 4 (finite element analysis) only where complicated loadings and geometry render the other methods unusable. Use elastic analysis charts if boundary conditions and loadings are appropriate.

3.1.3 Distribution of loads on beams



SK 3/1 Distribution of load on beams (Method 2).

Method 1
BS 8110: Part 1: 1985, clause 3.5.3.7.[1]

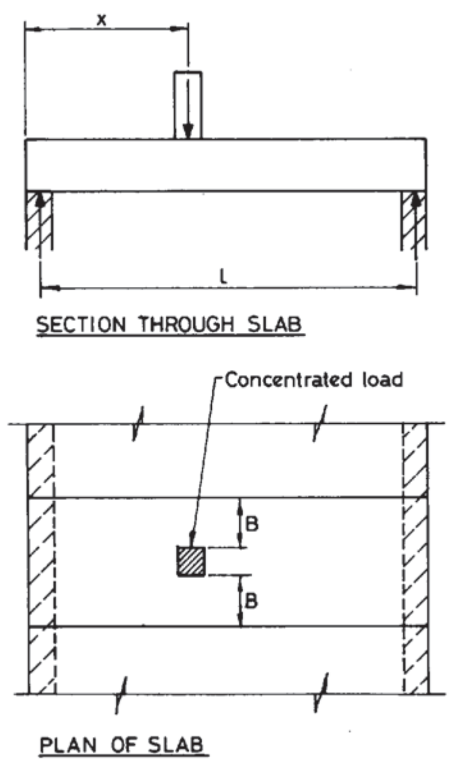
Method 2
Triangular and trapezoidal distribution of uniform load.

Method 3
Finite difference – Moody's Table.[9] Use the coefficients R_x and R_y to calculate distribution of loads on the edge beams.

Method 4
Finite element analysis. Use the support reactions as loading on the beam.

Recommendations
Method 2 may be used for all applications. Method 3 and Method 4 may be used when similar methods are used for the analysis of the slab panels.

3.1.4 Concentrated load on slab



SK 3/2 Effective width of slab to be considered for spread of a concentrated load on a simply supported one-way slab.

Simply supported slabs spanning in one direction only the width B on each side of load over which the load may be assumed to spread is given by:

$$B = 1.2x\left(1 - \frac{x}{l}\right)$$

where x = distance of load from support closest to load
l = effective span.

For slabs spanning in both directions published tables and charts should be used to find bending moment and shear per unit width of slab. A finite element model may be created to analyse a complicated loading arrangement.

3.2 LOAD COMBINATIONS

3.2.1 General rules

See Section 2.2.1.

3.2.2 Rules of load combination for continuous one-way spanning slab panels

See Section 2.2.2.

3.2.3 Redistribution of moments

See Section 2.2.3.

3.2.3.1 Two-way spanning slab panels

No redistribution is allowed when Method 1 or Method 2 of analysis in Section 3.1.2 is followed. Redistribution of 10% may be allowed when Method 3 or Method 4 is adopted. Note that reduction of support moments means a corresponding increase in span moment.

3.2.4 Exceptional loads

See Section 2.2.4.

3.3 STEP-BY-STEP DESIGN PROCEDURE FOR SLABS

Step 1 Analysis

Carry out analysis (follow Section 3.1.2).

Note: One-way spanning slabs should be treated as beams of unit width and Chapter 2 should be followed except for minimum shear reinforcement.

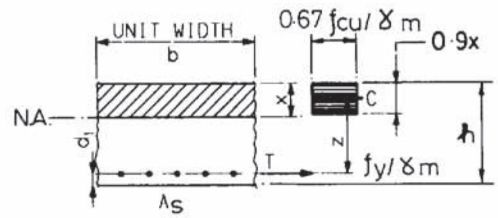
Step 2 Design forces

Draw panel of slab and indicate maximum design moments, shears and in-plane loads, if any, per unit width of slab.

Step 3 Cover to reinforcement

Determine cover required to reinforcement as per Tables 11.6 and 11.7. Find effective depth d, assuming reinforcement diameter. Use actual effective depth in each direction.

Step 4 Design of slab for flexure



SK 3/3 Section through slab showing stress due to moment.

Find the following parameters for design moments in Step 2 per unit width of slab.

$$M_d = M + N\left(\frac{h}{2} - d_1\right) \qquad \text{for } N \leq 0.1f_{cu}bd$$

Note: For $N > 0.1f_{cu}bd$, design as wall (see Chapter 8). M_d may also be taken equal to M where $N \leq 0.1f_{cu}bd$ and N may be ignored. (Sign convention: N is +ve for compression.)

$$K = \frac{M_d}{f_{cu}bd^2}$$

$$z = d\left[0.5 + \sqrt{\left(0.25 - \frac{K}{0.9}\right)}\right] \leq 0.95d$$

$$x = \frac{d - z}{0.45}$$

$$A_s = \frac{M_d}{0.87f_y z} - \frac{N}{0.87f_y}$$

$K' = 0.156$ when redistribution does not exceed 10%

$K' = 0.402(\beta_b - 0.4) - 0.18(\beta_b - 0.4)^2$ when redistribution exceeds 10%

$$\beta_b = \frac{M}{M'} < 0.9$$

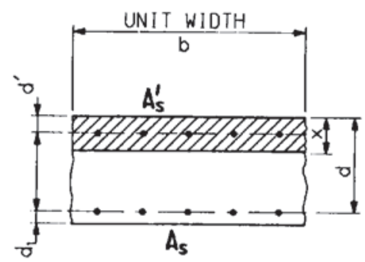
where M = moment after redistribution

M' = moment before redistribution.

Note: If K is greater than K', increase depth of slab and start from Step 1 unless links are provided in the zone where steel in compression is used. The links are required to provide lateral restraint to bars in compression. Links in slab should normally be avoided.

When $K > K'$,

$$z = d\left[0.5 + \sqrt{\left(0.25 - \frac{K'}{0.9}\right)}\right]$$



SK 3/4 Section through doubly reinforced slab.

$$x = \frac{d - z}{0.45} \leq 0.5d$$

$$A'_s = \frac{(K - K')f_{cu}bd^2}{f'_y(d - d')}$$

$$A_s = \frac{K'f_{cu}bd^2}{0.87f_y z} + A'_s - \frac{N}{0.87f_y}$$

If $d'/x > 0.43x$,

$$A'_s = \frac{(K - K')f_{cu}bd^2}{f'_s(d - d')}$$

$$A_s = \frac{K'f_{cu}bd^2}{0.87f_y z} + A'_s - \frac{N}{0.87f_y}$$

$$f'_s = \left(\frac{x - d'}{0.57x}\right)\varepsilon_y E_s \qquad \text{because steel strain } \varepsilon'_s = \left(\frac{x - d'}{0.57x}\right)\varepsilon_y$$

$$\varepsilon_y = \left(\frac{f_y}{\gamma_m}\right)E_s$$

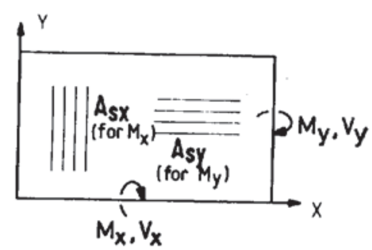
where ε_y corresponds to steel stress f_y/γ_m, as in Section 1.4.2.

Note: Follow detailing rules in clause 3.5.3.5 of BS 8110: Part 1: 1985[1] if analysis has been carried out using Table 3.15 of BS 8110. Design charts in BS 8110: Part 3: 1985 may be used.

Step 5 Detailing

Convert areas of steel per unit width found in Step 4 to diameter and spacing of bars.

Step 6 Check shear



SK 3/5 Plan of a panel of slab showing direction of reinforcement.

Find the following parameters at critical sections for shear.

$$v_x = \frac{V_x}{bd} \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$$

$$v_y = \frac{V_y}{bd} \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$$

$$p_x = \frac{100A_{sx}}{bd}$$

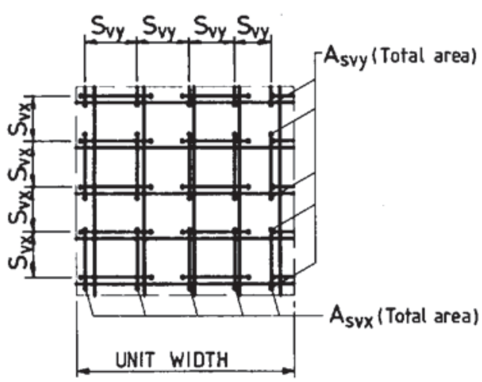
$$p_y = \frac{100A_{sy}}{bd}$$

Find v_{cx} and v_{cy} from Figs 11.2 to 11.5, depending on strength of concrete.

If $v_x < v_{cx}$ and $v_y < v_{cy}$, no shear reinforcement is required.

If $v_{cx} < v_x \leq (v_{cx} + 0.4)$ or $v_{cy} < v_y \leq (v_{cy} + 0.4)$, nominal links are required in the zone where v_x or v_y is greater than v_{cx} or v_{cy} respectively.

Find nominal links:



SK 3/6 Plan of unit area of slab showing shear reinforcement by links.

$$A_{svx} \geq \frac{0.4bS_{vx}}{0.87f_y} \quad \text{or} \quad A_{svy} \geq \frac{0.4bS_{vy}}{0.87f_y}$$

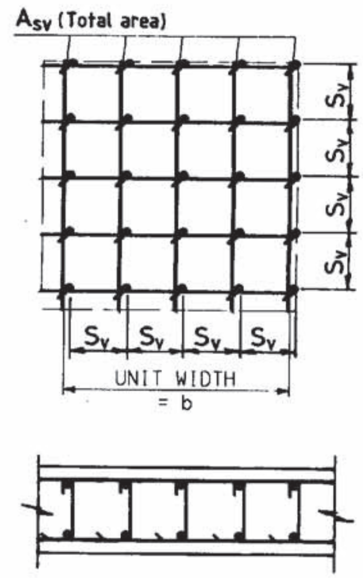
Note: Single vertical bars may be used instead of closed links provided proper anchorage bond length is available.

If $v_{cx} < v_x < (v_{cx} + 0.4)$
and $v_{cy} < v_y < (v_{cy} + 0.4)$

nominal links in both directions are required.

Assume $S_{vx} = S_{vy} = S_v$,

$$A_{sv} = \frac{0.8b\ S_v}{0.87f_y}$$

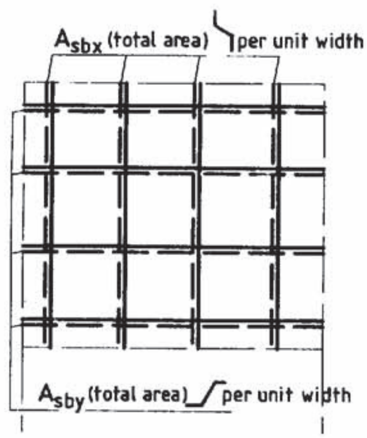


SK 3/7 Plan of unit area and section showing shear reinforcement by single vertical bars.

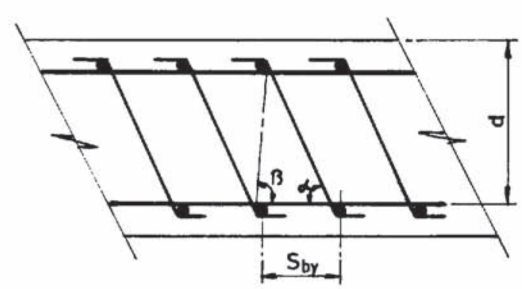
Provide single vertical bars with proper anchorage over the whole zone at a grid spacing of S_v.

If $(v_{cx} + 0.4) < v_x \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$
or $(v_{cy} + 0.4) < v_y \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$

use links or bent-up bars.



SK 3/8 Plan of slab showing bent-up bars as shear reinforcement.



SK 3/9 Section through slab showing bent-up shear reinforcement.

$$A_{svx} \geq \frac{bS_{vx}(v_x - v_{cx})}{0.87f_y} \quad \text{when using links for } V_x\text{, or}$$

$$A_{svy} \geq \frac{bS_{vy}(v_y - v_{cy})}{0.87f_y} \quad \text{when using links for } V_y\text{, or}$$

$$A_{sbx} \geq \frac{bdS_{bx}(v_x - v_{cx})}{0.87f_y(\cos\alpha + \sin\alpha\cot\beta)(d - d')}$$

when using bent-up bars for V_x, or

$$A_{sby} \geq \frac{bdS_{by}(v_y - v_{cy})}{0.87f_y(\cos\alpha + \sin\alpha\cot\beta)(d - d')}$$

when using bent-up bars for V_y

If $(v_{cx} + 0.4) < v_x \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$
and $(v_{cy} + 0.4) < v_y \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$

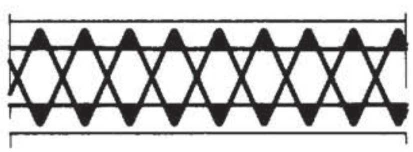
use bent-up bars in two orthogonal directions.

$$A_{sbx} \geq \frac{bdS_{bx}(v_x - v_{cx})}{0.87f_y\,(\cos\alpha + \sin\alpha\cot\beta)\,(d - d')}$$

and $$A_{sby} \geq \frac{bdS_{by}(v_y - v_{cy})}{0.87f_y\,(\cos\alpha + \sin\alpha\cot\beta)\,(d - d')}$$

Note: A_{sbx} and A_{sby} are the areas of bent-up bar required per unit width of slab equal to b.

Recommendation
Avoid using links or bent-up bars in slabs to resist shear. No shear reinforcement should be used in slabs up to 200 mm thick.



SK 3/10 Lacing system of shear reinforcement in slab.

A lacing system of shear reinforcement in slabs provided by bent-up bars at 45° to the tensile reinforcement works well where shear reinforcement and general increase of ductility are required. In this system, angles α and β may both be taken equal to 45°. In the formula for calculating the area of the bent-up bars, S_{bx} and S_{by} may be limited to $1.5d$.

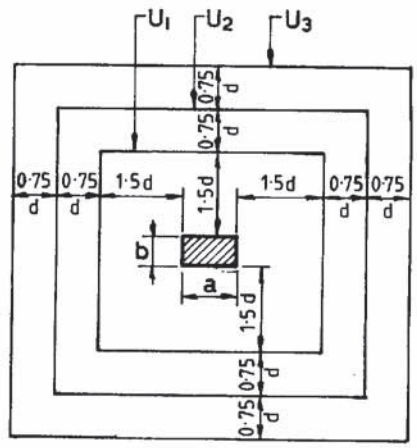
Step 7 Check punching shear

Check punching shear stress.

$$v_{max} = \frac{V}{U_{od}} \leq 0.8\,\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$$

where $U_o = 2(a + b)$ for rectangular load, or
$= $ perimeter of loaded area.

$$v_1 = \frac{V}{U_1 d}$$

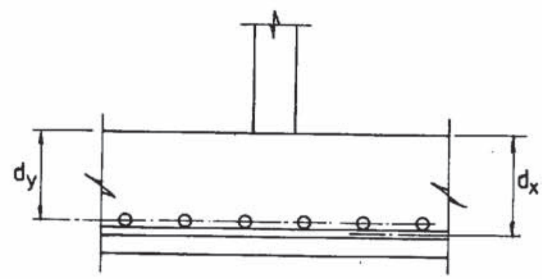


SK 3/11 Plan of slab around a concentrated load showing successive perimeters for punching shear check.

where $U_1 = 2(a + b + 6d)$ for rectangular loaded area, or
$= $ perimeter at $1.5d$ from face of loaded area.

$v_c =$ design concrete shear stress from Figs 11.2 to 11.5.

$V =$ concentrated load on slab



SK 3/12 Section through slab showing effective depths.

Calculate $p = 100\ A_s/bd$ under concentrated load to find v_c.

Note: Take p as the average of p_x and p_y where $p_x = 100A_{sx}/bd_x$ and $p_y = 100A_{sy}/bd_y$.

Shear reinforcement in first failure zone

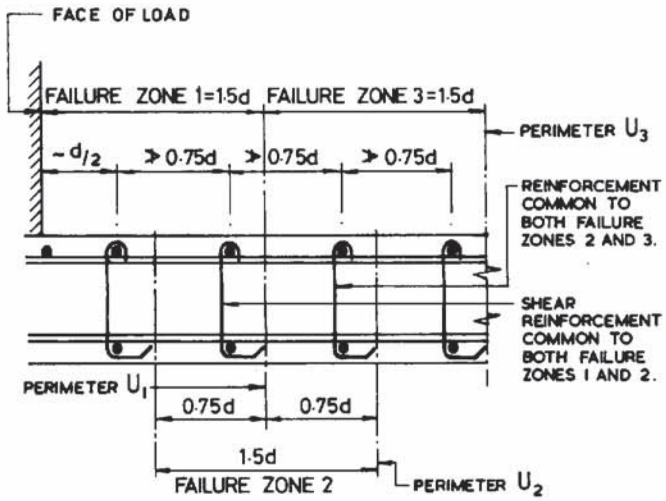
If $v_1 \leq v_c$, no shear reinforcement is required and no further checks are necessary.

If $v_1 \leq 1.6\ v_c$,

$$A_{sv}\ \sin\alpha \geq \frac{(v_1 - v_c)\ U_1 d}{0.87 f_y} \geq \frac{0.4 U_1 d}{0.87 f_y}$$

If $1.6 v_c < v_1 \leq 2\ v_c$,

$$A_{sv}\ \sin\alpha \geq \frac{5(0.7 v_1 - v_c)\ U_1 d}{0.87 f_y} \geq \frac{0.4 U_1 d}{0.87 f_y}$$



SK 3/13 Typical shear reinforcement for concentrated load on slab.

where A_{sv} is summation of areas of all shear reinforcement in a failure zone and α is the angle between the shear reinforcement and the plane of the slab. If v is greater than $2v_c$ then redesign slab with increased thickness or increased tensile steel or a combination of these parameters. It has been observed in tests that shear reinforcement in slabs does not work effectively if $v > 2v_c$.

Shear reinforcement in second failure zone

$$v_2 = \frac{V}{U_2 d}$$

where $U_2 = 2\,(a + b + 9d)$ for rectangular loaded area, or
$= $ perimeter at $2.25d$ from face of loaded area.

If $v_2 \le v_c$, no shear reinforcement is required and no further checks are necessary.

If $v_2 \le 1.6\ v_c$,

$$A_{sv}\ \sin\alpha \ge \frac{(v_2 - v_c)U_2 d}{0.87 f_y} \ge \frac{0.4U_2 d}{0.87 f_y}$$

If $1.6\ v_c < v_2 \le 2\ v_c$,

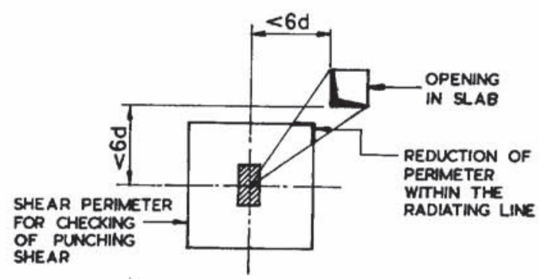
$$A_{sv}\ \sin\alpha \ge \frac{5(0.7v_2 - v_c)U_2 d}{0.87 f_y} \ge \frac{0.4U_2 d}{0.87 f_y}$$

Similarly check successive failure zones $0.75d$ apart till $v \le v_c$ is satisfied. Reinforcement to resist shear will be provided on at least two perimeters within a failure zone. Spacing of shear reinforcement on the perimeter should not exceed $1.5d$.

Steps to be followed for the determination of punching shear reinforcement in slabs

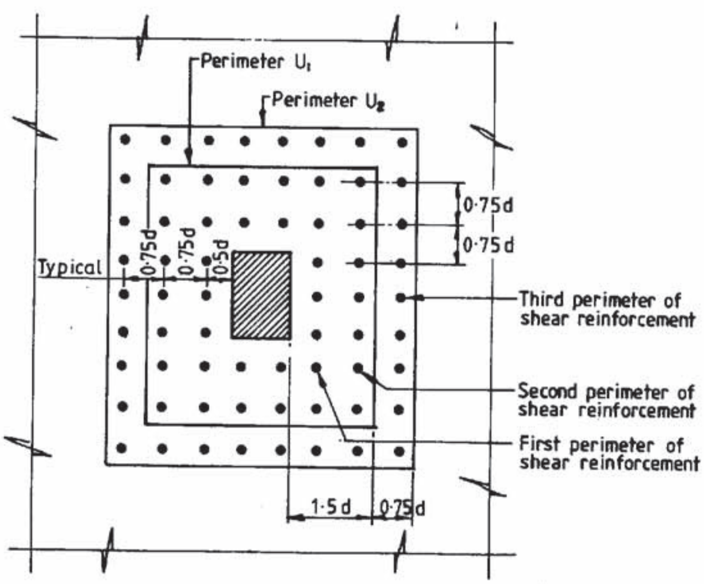
(1) The first failure zone is from the face of the loaded area to the perimeter 1.5d away.
(2) The first perimeter of shear reinforcement should be placed at $d/2$ from the face of the loaded area.
(3) The second perimeter of shear reinforcement should be placed at 0.75d from the first perimeter of shear reinforcement.
(4) A_{sv} is the sum of areas of all the legs of shear reinforcement in a failure zone in the first and second perimeter.
(5) The second failure zone is 1.5d wide and starts at 0.75d from the face of the loaded area.
(6) The successive failure zones are 1.5d wide and are 0.75d apart.
(7) The first perimeter reinforcement in the second failure zone is the same as the second perimeter reinforcement in the first failure zone.

Step 8 Modification due to holes



SK 3/14 Modification of shear perimeter due to presence of holes.

Carry out modification of U in Step 7 to allow for holes and proximity to edge.
The perimeter under consideration, U, in Step 7 will be reduced.



SK 3/15 Plan of slab near a concentrated load showing distribution of shear reinforcement.

Step 9 ***Minimum tension reinforcement***

$A_s \geq 0.0013bh$ in both directions

At end support of slabs where simple support has been assumed, provide in the top of slab half the area of bottom steel at midspan or $0.0013bh$, whichever is greater.

Step 10 ***Torsional reinforcement***

Special torsional reinforcement will be required at the corners of slab panels when the method of analysis follows clause 3.5.3.4 of BS 8110: Part 1: 1985. Follow clause 3.5.3.5 to determine the amount of torsional reinforcement.

Step 11 ***Check span/effective depth***

Find l_e/d, where l_e is the effective span in the shorter direction. Find basic span/effective depth ratio from Table 11.3.

Find service stress, $f_s = \left(\frac{5}{8\beta_b}\right)\left(\frac{A_{s\ reqd}}{A_{s\ prov}}\right)f_y$

where $\beta_b = M/M'$

M = moment after redistribution

M' = moment before redistribution.

Find M/bd^2.

Find modification factor for tension reinforcement from Chart 11.5 and modification factor for compression reinforcement from Chart 11.4.

Find modified span/depth ratio by multiplying the basic span/depth ratio with the modification factor for tensile reinforcement and compression reinforcement, if used.

Check $l_e/d <$ modified span/depth ratio.

Step 12 ***Curtailment of bars in tension***

Follow simplified detailing rules for slabs as in Fig. 3.34.

Step 13 ***Spacing of bars in tension***

Clear spacing of bars should not exceed $3d$ or 750 mm.

Percentage of reinforcement, $100\ A_s/bd$ (%)	Maximum clear spacing of bars in slabs (mm)
1 or over	160
0.75	210
0.5	320
0.3	530
less than 0.3	$3d$ or 750, whichever is less

A_s is the area required at the ultimate limit state. The clear spacings as given above may be multiplied by β_b to account for redistribution of moments. β_b is the ratio of moment after redistribution to moment before redistribution. These clear spacings deem to satisfy 0.3 mm crack width at serviceability limit state.

Step 14 ***Check early thermal cracking***

Early thermal cracking should be checked for the following pour configurations:

(1) Thin wall cast on massive base: $R = 0.6$ to 0.8 at base, $R = 0.1$ to 0.2 at top.
(2) Massive pour cast on blinding: $R = 0.1$ to 0.2.
(3) Massive pour cast on existing mass concrete: $R = 0.3$ to 0.4 at base, $R = 0.1$ to 0.2 at top.
(4) Suspended slabs: $R = 0.2$ to 0.4.
(5) Infill panels i.e. rigid restraint: $R = 0.8$ to 1.0.

where R = restraint factor

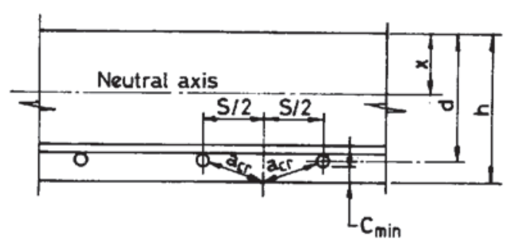
Typical values of T_1 for Ordinary Portland Cement (OPC) concrete are:

Section thickness (mm)	Steel formwork	Plywood formwork	Cast on ground
300	13°C	25°C	17°C
500	22°C	35°C	28°C
700	32°C	42°C	28°C
1000	42°C	47°C	28°C

These figures are based on average cement content of 350 kg/m^3.
Calculate:

$$\varepsilon_r = 0.8\ T_1\ \alpha\ R$$

obtain α (coefficient of thermal expansion) from Table 2.3 in Section 2.1.9.



SK 3/16 Section of slab for crack width calculation.

$$W_{max} = \frac{3a_{cr}\ \varepsilon_r}{1 + \dfrac{2(a_{cr} - C_{min})}{h - x}}$$

Assume $x = h/2$

Note: If W_{max} is greater than design crack width, which is normally taken equal to 0.3 mm, then suggest means for reducing T_1.

Step 15 Check minimum reinforcement to distribute early thermal cracking

$\rho_{crit} = 0.0035$ for Grade 460 steel reinforcement

$A_s = A_c\rho_{crit}$

For suspended slabs and walls,

$$A_c = \frac{bh}{2} \quad \text{or} \quad 250b \qquad \text{whichever is smaller}$$

$A_s = 0.0035\ A_c$ near each face in each direction of slab and wall

For ground slabs and foundation bases,
up to 300 mm thickness:

$A_{st} = 0.00175bh$ near top surface in each direction

from 300 mm to 500 mm thickness:

$A_{st} = 0.00175bh$ near top surface in each direction

$A_{sb} = 0.35b$ near bottom surface in each direction

over 500 mm thickness:

$A_{st} = 0.875b$ near top surface in each direction

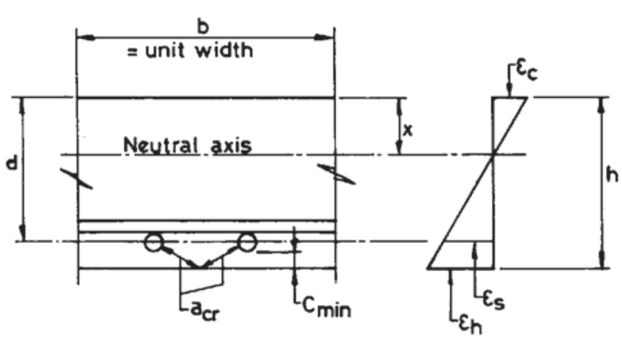
$A_{sb} = 0.35b$ near bottom surface in each direction

Step 16 Check flexural crack width

Serviceability limit state

$LC_7 = 1.0DL + 1.0LL + 1.0EP + 1.0WP + 1.0WL$

Note: Omit loadings from LC_7 which produce beneficial rather than adverse effect.



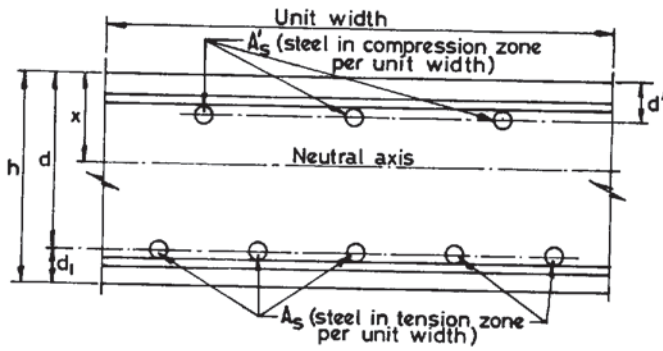
SK 3/17 Section through slab for the calculation of flexural crack width.

$$W_{max} = \frac{3a_{cr}\,\varepsilon_m}{1 + \dfrac{2(a_{cr} - C_{min})}{h - x}}$$

$$\varepsilon_{mh} = \varepsilon_h - \frac{b(h - x)^2}{3E_s A_s(d - x)}$$

Note: ε_h is the strain due to load combination LC_7 at depth h from compression face, b is the unit width of slab, and A_s is the area of tensile steel per unit width of slab.

For slab, b is taken equal to unit width.



SK 3/18 Section of slab with steel in compression zone.

$$m = \frac{E_s}{E_c} \qquad p = \frac{A_s}{bd} \qquad p' = \frac{A_s'}{bd}$$

$$x = d\left\{\left[(mp + (m - 1)p')^2 + 2\left(mp + (m - 1)\left(\frac{d'}{d}\right)p'\right)\right]^{\frac{1}{2}} - (mp + (m - 1)p')\right\}$$

$$k_2 = \left(\frac{x}{2d}\right)\left(1 - \frac{x}{3d}\right)$$

$$k_3 = (m - 1)\left(1 - \frac{d'}{x}\right)$$

$$f_c = \frac{M}{k_2 bd^2 + k_3 A_s'(d - d')}$$

$$f_s = mf_c\left(\frac{d}{x} - 1\right)$$

$$\varepsilon_s = \frac{f_s}{E_s}$$

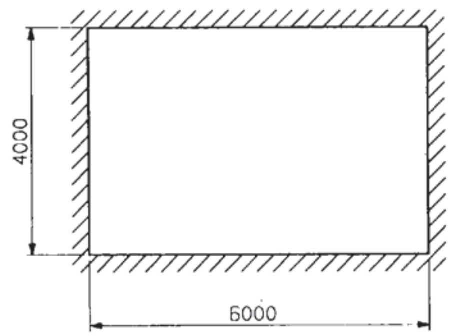
$$\varepsilon_h = \left(\frac{h - x}{d - x}\right)\varepsilon_s$$

$$\varepsilon_{mh} = \varepsilon_h - \frac{b(h - x)^2}{3E_s A_s(d - x)}$$

Note: In normal internal or external condition of exposure where the limitation of crack widths to 0.3 mm is appropriate, Step 13 will deem to satisfy the crack width criteria.

3.4 WORKED EXAMPLE

Example 3.1 Design of a two-way slab panel



SK 3/19 Plan of a panel of slab continuous on all sides.

Clear panel size is 6 m × 4 m
Thickness of slab = 150 mm
Imposed loading = 20 kN/m^2
Finishes = 2 kN/m^2
Panel of slab continuous on all four sides
Width of beam = 300 mm

Step 1 Analysis of slab panel

Effective span, $l_e = l_o$

$$l_x = 4.3\,\text{m}$$
$$l_y = 6.3\,\text{m}$$

$$l_x/l_y = 0.68$$

Elastic analysis

Read coefficients from Fig. 3.12:

$$m_{x1} = 0.035$$
$$m_{y1} = 0.021$$
$$m'_{x2} = 0.075$$
$$m'_{y2} = 0.060$$

Characteristic dead load = 0.15 m × 25 kN/m^3 × 1.4 + 2 × 1.4
= 8.0 kN/m^2

Characteristic imposed load $= 1.6 \times 20 = 32\,\text{kN/m}^2$

n = ultimate load on slab $= 8 + 32 = 40\,\text{kN/m}^2$

$$M_{x1} = m_{x1} n l_x^2$$
$$= 0.035 \times 40 \times 4.3^2$$
$$= 25.9\,\text{kNm/m}$$

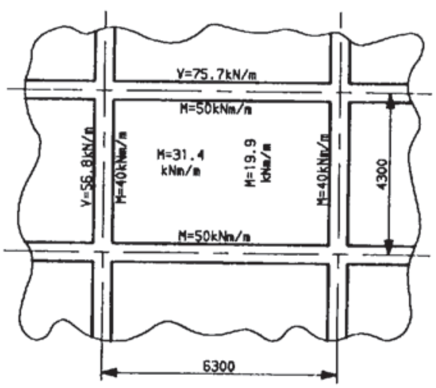
$$M_{y1} = 0.021 \times 40 \times 4.3^2$$
$$= 15.5\,\text{kNm/m}$$

$$M'_{x3} = 0.075 \times 40 \times 4.3^2$$
$$= -55.5\,\text{kNm/m}$$

$$M'_{y2} = 0.060 \times 40 \times 4.3^2$$
$$= -44.4\,\text{kNm/m}$$

Allowing for 10% redistribution of moments,
Design moments:

$M_{x1} = 31.4\,\text{kNm/m}$
$M_{y1} = 19.9\,\text{kNm/m}$
$M'_{x3} = -50.0\,\text{kNm/m}$
$M'_{y2} = -40.0\,\text{kNm/m}$



SK 3/20 Plan of panel of slab showing bending moments and shears.

Note: These moments do not take into account the Wood–Armer effect due to the presence of M_{xy} and may be unconservative locally. In ultimate load design local plastic hinge formation may be tolerated when there is a possibility of redistribution of loads.

Analysis following BS 8110: Part 1: 1985 [1]
Coefficients from Table 3.15.

Interior panel $l_y/l_x = 1.46$

$m_{sx1} = 0.039$ $\quad M_{x1} = 28.8\,\text{kNm/m}$
$m_{sy1} = 0.024$ $\quad M_{y1} = 17.8\,\text{kNm/m}$
$m'_{sx3} = 0.052$ $\quad M'_{x3} = 38.5\,\text{kNm/m}$
$m'_{sx2} = 0.032$ $\quad M'_{y2} = 23.7\,\text{kNm/m}$

Note: These moments are considerably less than the redistributed design moments found from elastic analysis. Elastic analysis gives peak values, whereas the BS 8110 coefficients tend to smear them across a long stretch of slab.

It is desirable and practical to use the elastic analysis results and allow 10% redistribution with a view to minimising the appearance of unsightly cracks in the slab. This is a conservative approach.

Check by yield-lines analysis

Assume that the elastic analysis moments are ultimate capacity moments in the panel of slab.

$M_{VN} = 50\,\text{kNm/m}$ (Vertical Negative)
$M_{VP} = 31.4\,\text{kNm/m}$ (Vertical Positive)
$M_{HN} = 40\,\text{kNm/m}$ (Horizontal Negative)
$M_{HP} = 19.9\,\text{kNm/m}$ (Horizontal Positive)

Assume that the elastic analysis results will be the maximum plastic moments in the panel of slab.

$$\frac{L}{H}\left(\frac{M_{VN} + M_{VP}}{M_{HN} + M_{HP}}\right)^{\frac{1}{2}} = \frac{6.3}{4.3}\left(\frac{50 + 31.4}{40 + 19.9}\right)^{\frac{1}{2}}$$
$$= 1.70$$

Assume symmetrical yield-lines – see Table 3.2.
Refer to appropriate diagram from Figs 3.18 to 3.33.
Refer to Fig. 3.22 and find x/L

$$\frac{x}{L} = 0.35$$
$$x = 0.35 \times 6.3 = 2.20\,\text{m}$$

Unit resistance, $r = \dfrac{5(M_{HN} + M_{HP})}{x^2}$ from Table 3.2

$$= \frac{5 \times 59.9}{2.2^2} = 61.9\,\text{kN/m}^2 > 40\,\text{kN/m}^2$$

Alternatively,

$r = \dfrac{8(M_{VN} + M_{VP})(3L - x)}{H^2(3L - 4x)}$ from Table 3.2

$$= \frac{8(50 + 31.4)(3 \times 6.3 - 2.2)}{4.3^2(3 \times 6.3 - 4 \times 2.2)}$$
$$= 58.23\,\text{kN/m}^2 > 40\,\text{kN/m}^2$$

Note: The values of M_{VN}, M_{VP}, M_{HN} and M_{HP} could be readjusted to arrive at r as close to $40\,\text{kN/m}^2$ as possible.

Designed by the results of elastic analysis the slab panel has a large reserve of strength because the failure loading is $58.23\,\text{kN/m}^2$ against design ultimate loading of $40\,\text{kN/m}^2$. Similarly, designed by the results of the BS 8110 method of analysis, the panel of slab has a small reserve of strength because the calculated collapse loading is 46.3 kN/m^2.

To check crack widths and deflection due to service load the BS 8110 coefficients may not be used. Always use the elastic analysis results.

Determination of shear at supports
Use BS 8110: Part 1: 1985, Table 3.16.[1]

Shear coefficients 0.44 and 0.33

$$V_x = 0.44 \times 40 \times 4.3 = 75.7\,\text{kN/m}$$

$$V_y = 0.33 \times 40 \times 4.3 = 56.8\,\text{kN/m}$$

Refer to Table 3.4.

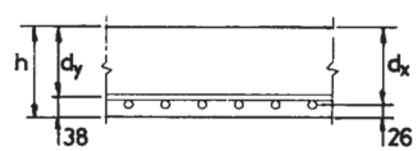
By yield-line principle: assuming $r = 40\,\text{kN/m}^2$,

$$V_x = \frac{3\,r\,H\,(1 - x/L)}{2(3 - x/L)} = \frac{3 \times 40 \times 4.3 \times (1 - 0.35)}{2(3 - 0.35)} = 63.3\,\text{kN/m}$$

$$V_y = \frac{3\,r\,x}{5} = \frac{3 \times 40 \times 2.2}{5} = 52.8\,\text{kN/m}$$

Step 2 Draw diagram of panel of slab
See diagram with moments and shears marked on the panel (in Step 1).

Step 3 Determination of cover
Assume diameter of main reinforcement = 12 mm
Maximum size of aggregate = 20 mm
Condition of exposure = mild
Grade of concrete = C40
Minimum cement content = 325 kg/m^3
Maximum free water/cement ratio = 0.55
Fire resistance required = 1 hour
Nominal cover, as per Tables 11.6 and 11.7 = 20 mm
Effective depth, $d_x = 150 - 20 - 6 = 124$ mm
Effective depth, $d_y = 150 - 20 - 12 - 6 = 112$ mm



SK 3/21 Section through slab showing effective depths.

Step 4 Design of slab

Over continuous long edge, $M = 50\,\text{kNm/m}$

$$K = \frac{M}{f_{cu}bd_x^2} = \frac{50 \times 10^6}{40 \times 1000 \times 124^2} = 0.081$$

$$z = d\left[0.5 + \sqrt{\left(0.25 - \frac{K}{0.9}\right)}\right] = 0.9d = 111.6\,\text{mm}$$

$$x = \frac{d - z}{0.45} = 27.5\,\text{mm}$$

$$A_s = \frac{M}{0.87f_yz} = \frac{50 \times 10^6}{0.87 \times 460 \times 111.6} = 1120\,\text{mm}^2/\text{m}$$

Over continuous short edge, $M = 40\,\text{kNm/m}$

$$K = \frac{M}{f_{cu}bd_y^2} = \frac{40 \times 10^6}{40 \times 1000 \times 112^2} = 0.08$$

$$z = 0.9d = 100.8\,\text{mm}$$

$$x = 24.9\,\text{mm}$$

$$A_s = 992\,\text{mm}^2/\text{m}$$

Positive midspan moment in short direction

$$M = 31.4\,\text{kNm/m}$$

$$K = 0.051$$

$$z = 116.5\,\text{mm}$$

$$A_s = 673\,\text{mm}^2/\text{m}$$

Positive midspan moment in long direction

$$M = 19.9\,\text{kNm/m}$$

$$K = 0.04$$

$$z = 0.95d = 106.4\,\text{mm}$$

$$A_s = 467\,\text{mm}^2/\text{m}$$

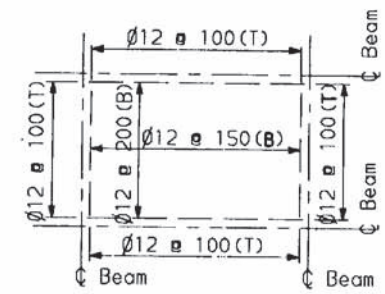
Step 5 Diameter and spacing of bars

Use:

Over long edge 12 dia. at 100 centre-to-centre (top) (1131 mm^2/m)

Over short edge 12 dia. at 100 centre-to-centre (top) (1131 mm^2/m)

Short direction at midspan 12 dia. at 150 centre-to-centre (bottom) (754 mm^2/m)



SK 3/22 Plan of panel of slab showing design steel requirement.

Long direction at midspan 12 dia. at 200 centre-to-centre (bottom) ($565\,\text{mm}^2/\text{m}$)

Step 6 ***Check shear stress***

$$v_x = \frac{V_x}{bd_x} = \frac{75.7 \times 10^3}{1000 \times 124} = 0.61\,\text{N/mm}^2$$

$$v_y = \frac{V_y}{bd_y} = \frac{56.8 \times 10^3}{1000 \times 112} = 0.51\,\text{N/mm}^2$$

$$p_x = \frac{100A_{sx}}{bd_x} = \frac{100 \times 1131}{1000 \times 124} = 0.91\%$$

$$p_y = \frac{100A_{sy}}{bd_y} = \frac{100 \times 1131}{1000 \times 112} = 1.0\%$$

From Fig. 11.5,

$$v_{cx} = 0.97\,\text{N/mm}^2 > v_x = 0.61\,\text{N/mm}^2$$

No shear reinforcement required.

Step 7 ***Check punching shear stress***
Not required.

Step 8 ***Modification due to holes***
Not required.

Step 9 ***Minimum tension reinforcement***

$$\begin{aligned} A_s &= 0.0013bh \\ &= 0.0013 \times 1000 \times 150 \\ &= 195\,\text{mm}^2/\text{m} \quad \text{satisfied} \end{aligned}$$

Step 10 ***Torsional reinforcement***
Not required.

Step 11 Check span/effective depth

$$\frac{l_{cx}}{d_x} = \frac{4.3 \times 10^3}{124}$$
$$= 34.7$$

Basic span/effective depth ratio = 26 from Table 11.3

$$\beta_b = \frac{M'}{M} = \frac{31.4}{25.9} = 1.21$$

where M' = moment after redistribution; M = moment before redistribution

$$f_s = \frac{5}{8} f_y \left(\frac{A_{s\ reqd}}{A_{s\ prov}}\right)\left(\frac{1}{\beta_b}\right)$$
$$= \frac{5}{8} \times 460 \times \frac{673}{754} \times \frac{1}{1.21}$$
$$= 212\,\text{N/mm}^2$$

$$\frac{M}{bd^2} = \frac{31.4 \times 10^6}{1000 \times 124^2} = 2.0$$

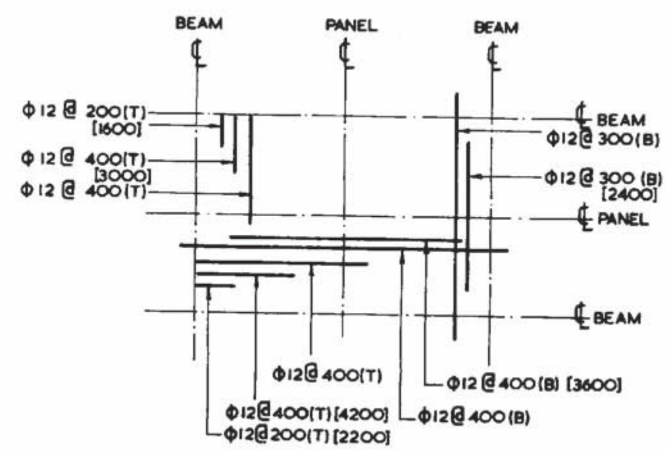
From Chart 11.5,

modification factor = 1.33

Modified span/effective depth ratio = 26 × 1.33 = 34.58 < 34.7

Code deflection limits have been exceeded slightly.
May be ignored.

Step 12 Curtailment of bars



SK 3/23 Plan of panel of slab showing arrangement of reinforcement.

45 × dia. of bars = 45 × 12 = 540 mm

$0.15\ l_{ex} = 0.15 \times 4.3 = 645$ mm
$0.30\ l_{ex} = 0.30 \times 4.3 = 1290$ mm
$0.20\ l_{ex} = 0.20 \times 4.3 = 860$ mm
$0.15\ l_{ey} = 0.15 \times 6.3 = 945$ mm
$0.30\ l_{ey} = 0.30 \times 6.3 = 1890$ mm
$0.20\ l_{ey} = 0.20 \times 6.3 = 1260$ mm

Direction l_x – top reinforcement

12 dia. @ 100 c/c to 800 mm from centre of beam (top)
12 dia. @ 200 c/c to 1500 mm from centre of beam (top)

Direction l_y – top reinforcement

12 dia. @ 100 c/c to 1100 mm from centre of beam (top)
12 dia. @ 200 c/c to 2100 mm from centre of beam (top)
Elsewhere use 12 dia. @ 400 c/c (top) both directions (282 mm^2)

Direction l_x – bottom reinforcement

12 dia. @ 150 c/c up to 800 mm from centre of beam (bottom)
12 dia. @ 300 c/c over beam (bottom)

Direction l_y – bottom reinforcement

12 dia. @ 200 c/c up to 1200 mm from centre of beam (bottom)
12 dia. @ 400 c/c over beam (bottom)

Step 13 Spacing of bars
Percentage of reinforcement in slab = 1%

Maximum clear spacing allowed = 160 mm

Actual spacing used = 100 mm OK

Maximum spacing of bars in tension = $3d = 3 \times 112 = 336$ mm

Maximum spacing used for designed bars in tension = 200 mm OK

Maximum spacing of nominal reinforcement to control early thermal cracking = 400 mm

Step 14 Check thermal cracking
For suspended slab, $R = 0.3$ assumed

T_1 = 12°C assumed for 150 mm thick slab

$\alpha = 12 \times 10^{-6}$ per degree C

$$\varepsilon_r = 0.8\ T_1\ \alpha\ R$$
$$= 0.8 \times 12 \times 12 \times 10^{-6} \times 0.3$$
$$= 34.56 \times 10^{-6}$$

$$C_{min} = 20\,mm + 12\,mm \quad \text{(dia. of bar)}$$
$$= 32\,mm \quad (\text{direction } l_y)$$

$$x = d/2 \text{ assumed} = 112/2 = 56\,mm \ (\text{direction } l_y)$$

$$a_{cr} = \sqrt{(200^2 + 38^2)} - 6 = 197.6\,mm$$

$$W_{max} = \frac{3a_{cr}\,\varepsilon_r}{1 + \dfrac{2(a_{cr} - C_{min})}{(h - x)}}$$

$$= \frac{3 \times 197.6 \times 34.56 \times 10^6}{1 + \dfrac{2(197.6 - 32)}{(150 - 56)}}$$

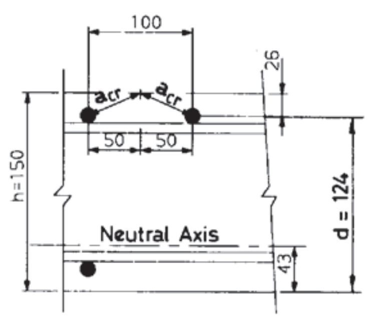
$$= 0.0045\,mm < 0.3\,mm \qquad \text{OK}$$

Step 15 Check minimum reinforcement to distribute cracking

$$A_c = \frac{bh}{2} = \frac{1000 \times 150}{2} = 75000\,mm^2$$

$$A_s = 0.0035\,A_c = 262.5\,mm^2/m$$
A_s provided = 12 dia. @ 400 c/c ($282\,mm^2/m$)

Step 16 Assessment of crack width in flexure



SK 3/24 Section through slab over beam for crack width calculations.

Service load on slab = $25.75\,kN/m^2$

By elastic analysis,

maximum bending moment over long support

$$= 0.075 \times 25.75 \times 4.3^2$$
$$= 35.7\,kNm/m$$

$$A_s = 1131\,mm^2/m \qquad A_s/bd = 9.12 \times 10^{-3}$$

$$b = 1000\,mm \qquad d = 124\,mm$$

$$m = 10 = E_s/E_c$$

$$A'_s = \text{neglected}$$

$$x = d[((mp)^2 + 2\,mp)^{\frac{1}{2}} - mp] = 43\,mm$$

$$z = d - \frac{x}{3} = 124 - \frac{43}{3} = 109.7\,\text{mm}$$

$$f_s = \frac{M}{A_s z} = \frac{35.7 \times 10^6}{1131 \times 109.7} = 288\,\text{N/mm}^2$$

$$\varepsilon_s = \frac{f_s}{E_s} = \frac{288}{200 \times 10^3} = 1.44 \times 10^{-3}$$

$$\varepsilon_h = \left(\frac{h - x}{d - x}\right) \varepsilon_s = \left(\frac{150 - 43}{124 - 43}\right) \times 1.44 \times 10^{-3} = 1.90 \times 10^{-3}$$

$$\varepsilon_{mh} = \varepsilon_h - \frac{b(h - x)^2}{3E_s A_s(d - x)} = 1.90 \times 10^{-3} - \frac{1000(150 - 43)^2}{3 \times 200 \times 10^3 \times 1131 \times (124 - 43)}$$

$$= 1.69 \times 10^{-3}$$

$$C_{min} = 20\,\text{mm}$$

$$a_{cr} = \sqrt{(26^2 + 50^2)} - 6 = 50\,\text{mm}$$

$$W_{cr} = \frac{3a_{cr}\,\varepsilon_m}{1 + \frac{2(a_{cr} - C_{min})}{(h - x)}} = 0.16\,\text{mm} < 0.3\,\text{mm} \qquad \text{OK}$$

3.5 FIGURES AND TABLES FOR CHAPTER 3

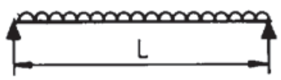
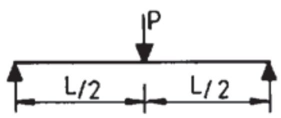
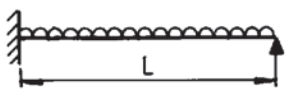
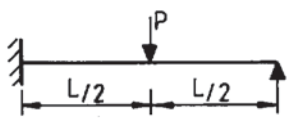
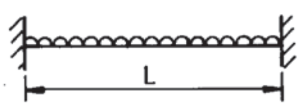
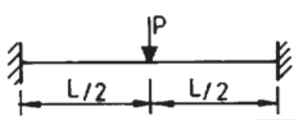
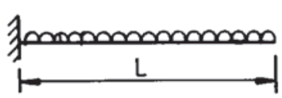
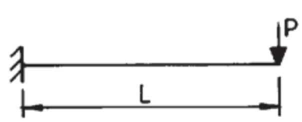
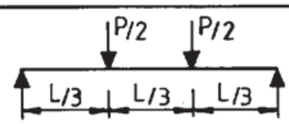
Edge conditions and loading diagrams	Elastic resistance, r_e	Elasto-plastic resistance, r_{ep}
	r_u	—
	R_u	—
	$\frac{8M_N}{L^2}$	r_u
	$\frac{16M_N}{3L}$	R_u
	$\frac{12M_N}{L^2}$	r_u
	$\frac{8M_N}{L}$	R_u
	r_u	—
	R_u	—
	R_u	—

Fig. 3.1 Elastic and elasto-plastic unit resistances for one-way elements.

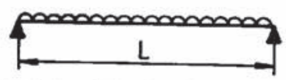
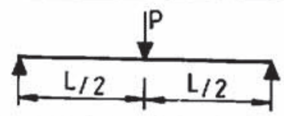
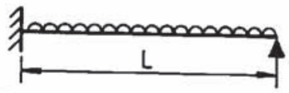
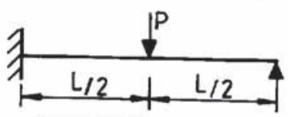
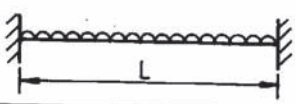
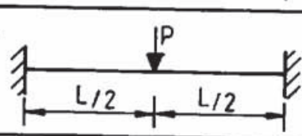
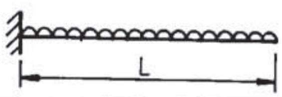
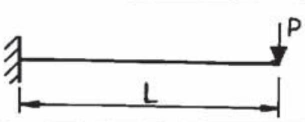
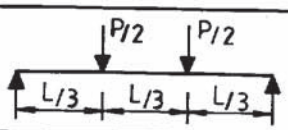
Edge conditions and loading diagrams	Support reactions, V_s
	$\frac{r_u L}{2}$
	$\frac{R_u}{2}$
	L. reaction $\frac{5r_uL}{8}$ R. reaction $\frac{3r_uL}{8}$
	L. reaction $\frac{11R_u}{16}$ R. reaction $\frac{5R_u}{16}$
	$\frac{r_uL}{2}$
	$\frac{R_u}{2}$
	r_uL
	R_u
	$\frac{R_u}{2}$

Fig. 3.2 Support shears for one-way elements (to be read in conjunction with Fig. 3.1).

Fig. 3.3 Moment and deflection coefficients for uniformly loaded two-way element with two adjacent edges fixed and two edges free.[8]

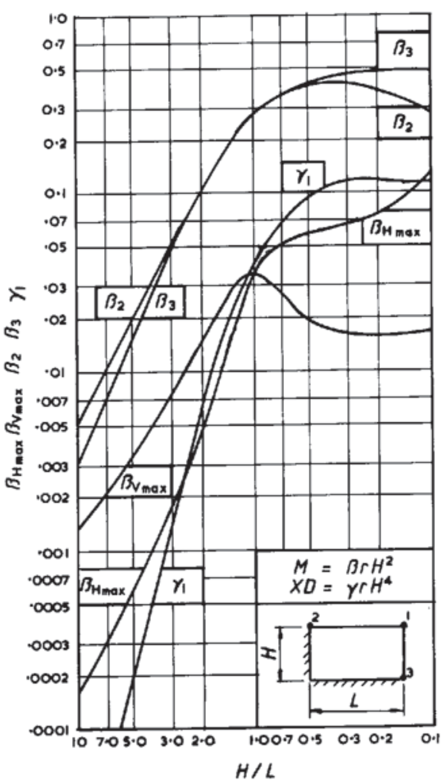


Fig. 3.4 Moment and deflection coefficients for uniformly loaded two-way element with one edge fixed, an adjacent edge simply supported and two edges free.[8]

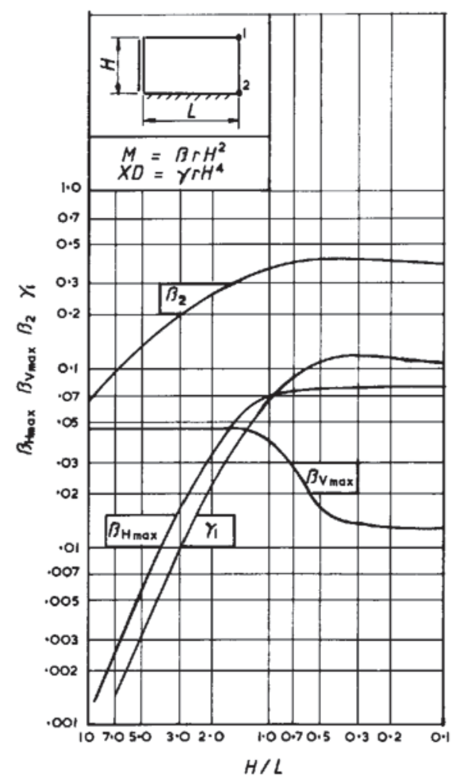


Fig. 3.5 Moment and deflection coefficients for uniformly loaded two-way element with two adjacent edges simply supported and two edges free.[8]

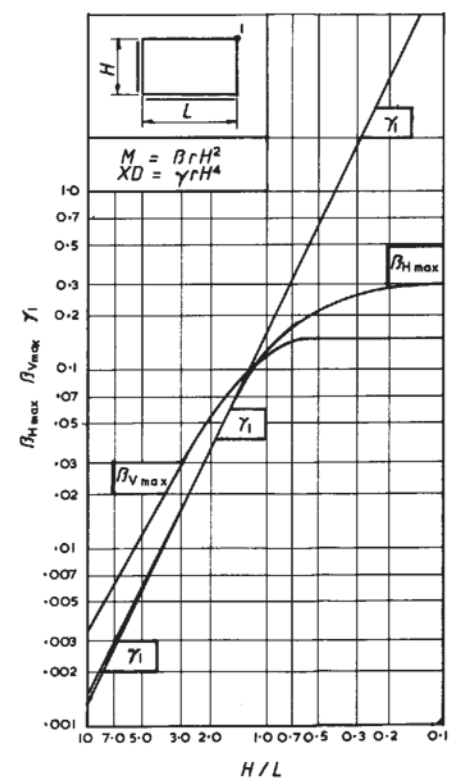


Fig. 3.6 Moment and deflection coefficients for uniformly loaded two-way element with three edges fixed and one edge free.[8]

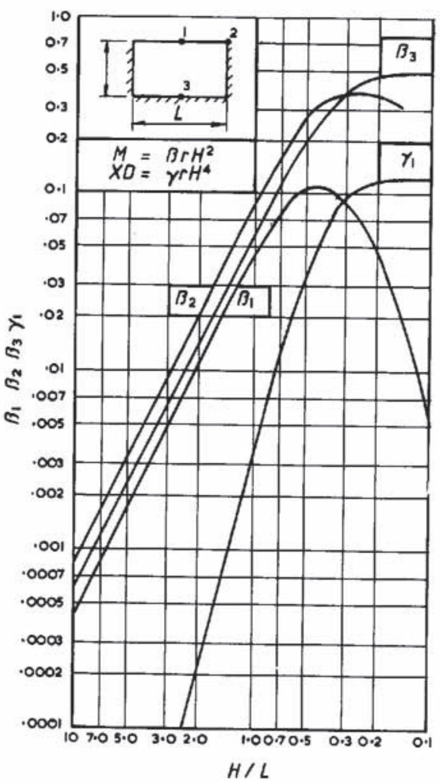


Fig. 3.7 Moment and deflection coefficients for uniformly loaded two-way element with two opposite edges fixed, one edge simply supported and one edge free.[8]

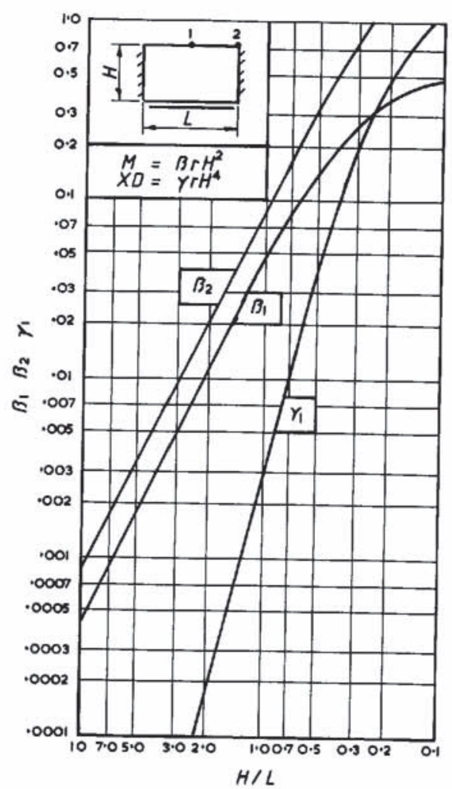


Fig. 3.8 Moment and deflection coefficients for uniformly loaded two-way element with two opposite edges simply supported, one edge fixed and one edge free.[8]

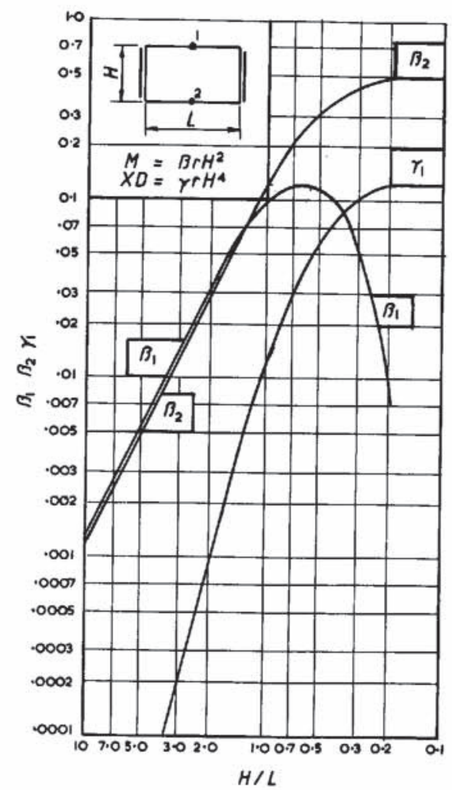


Fig. 3.9 Moment and deflection coefficients for uniformly loaded two-way element with three edges simply supported and one edge free.[8]

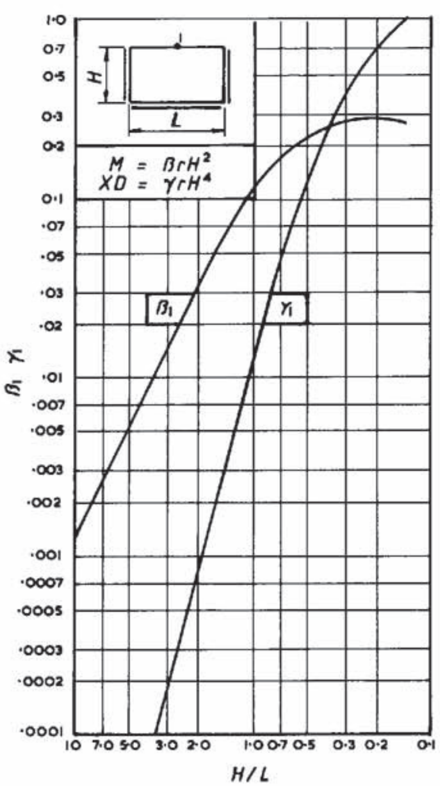


Fig. 3.10 Moment and deflection coefficients for uniformly loaded two-way element with two edges fixed, one edge simply supported and one edge free.[8]

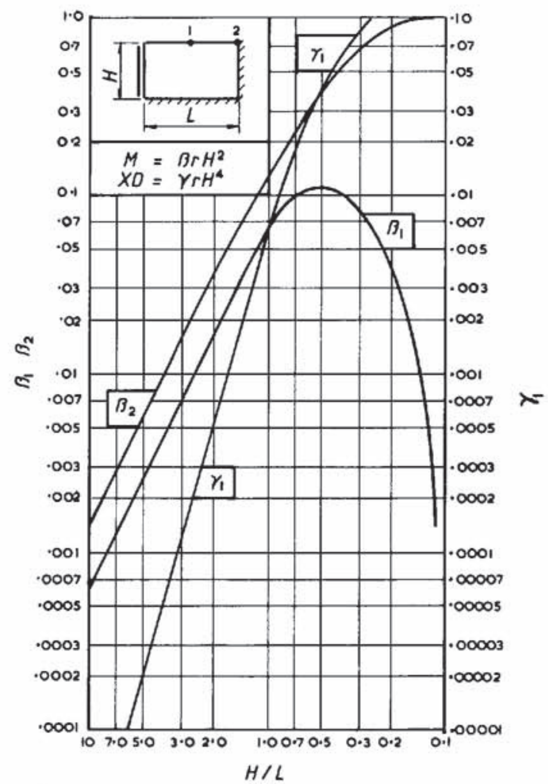


Fig. 3.11 Moment and deflection coefficients for uniformly loaded two-way element with two adjacent edges simply supported, one edge fixed and one edge free.[8]

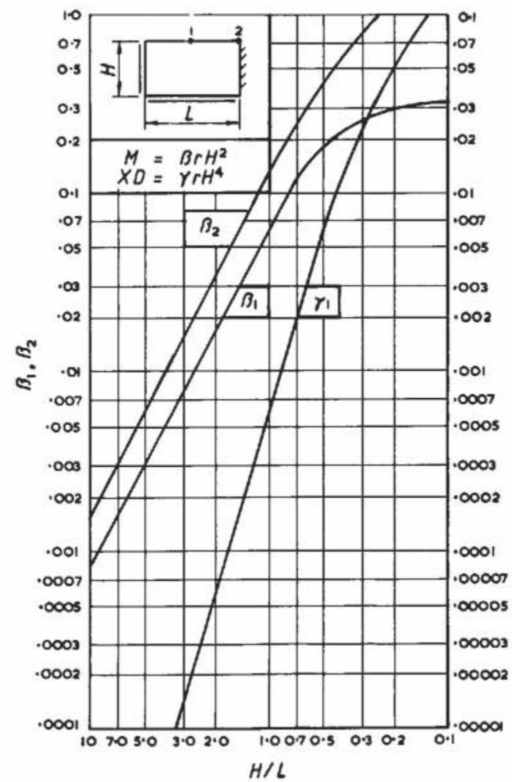


Fig. 3.12 Moment and deflection coefficients for uniformly loaded two-way element with all edges fixed.[8]

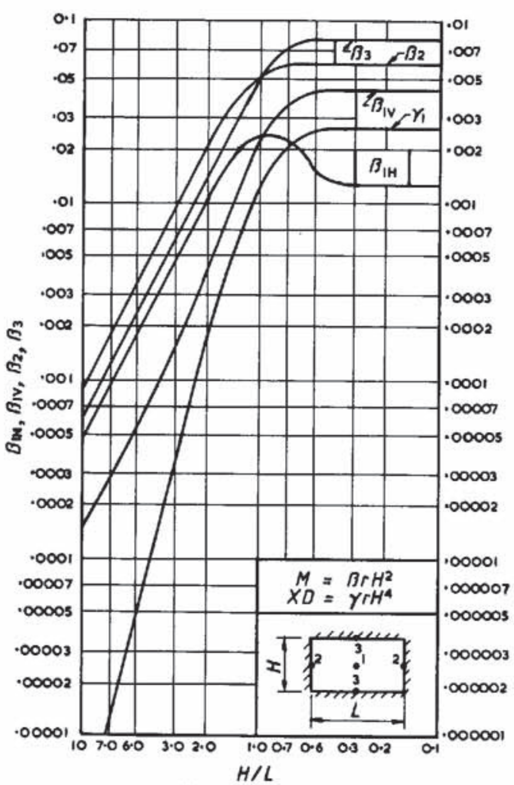


Fig. 3.13 Moment and deflection coefficients for uniformly loaded two-way element with two opposite edges fixed and two edges simply supported.[8]

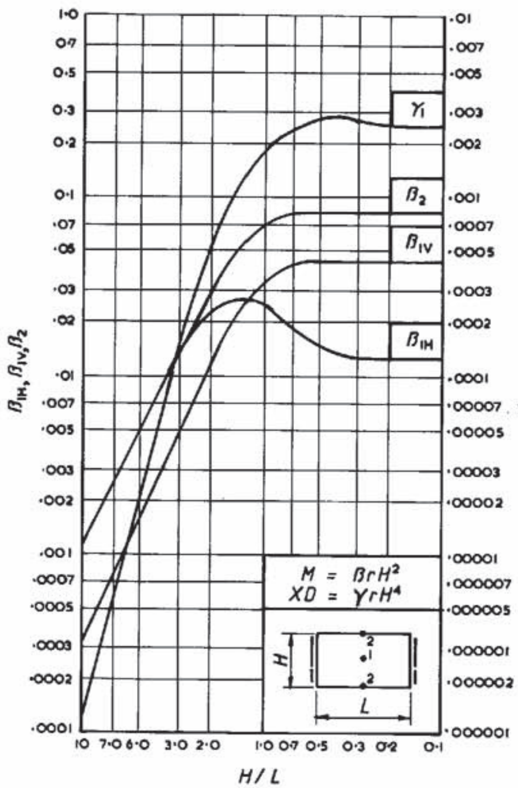
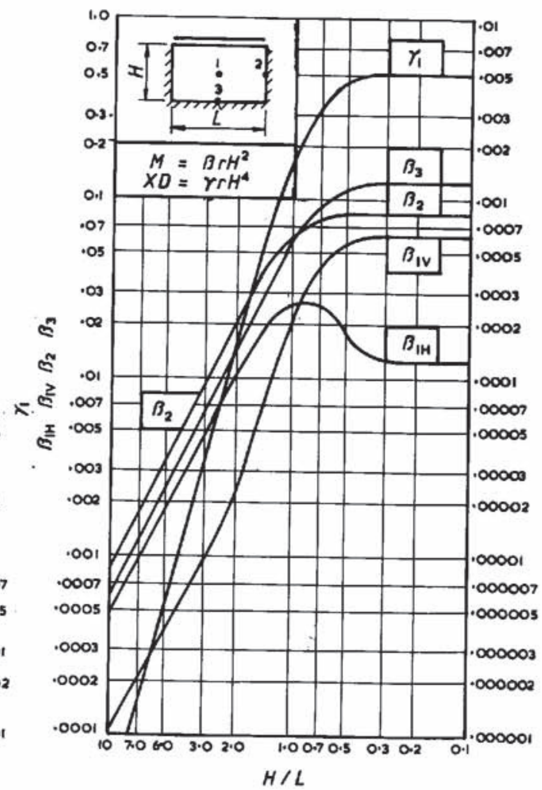


Fig. 3.14 Moment and deflection coefficients for uniformly loaded two-way element with three edges fixed and one edge simply supported.[8]



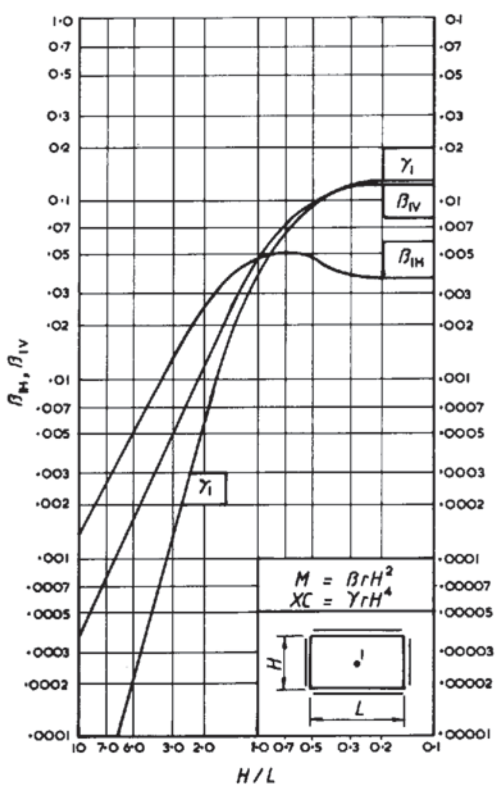


Fig. 3.15 Moment and deflection coefficients for uniformly loaded two-way element with all edges simply supported.[8]

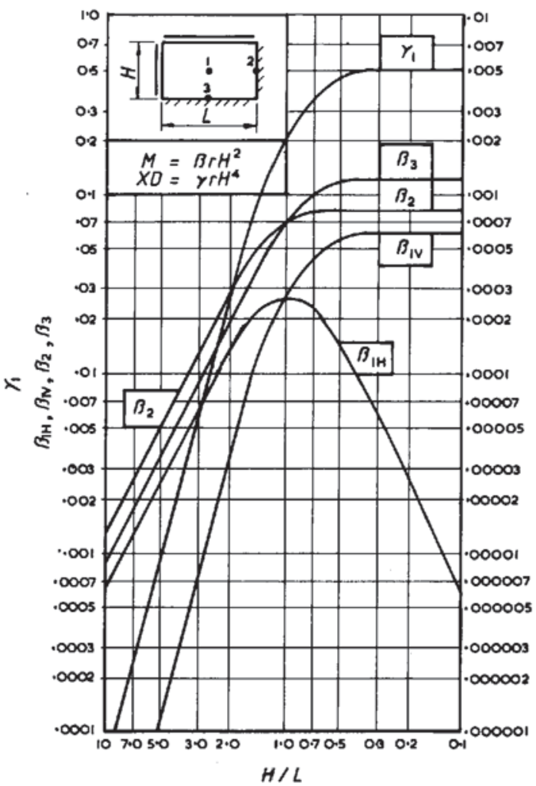


Fig. 3.16 Moment and deflection coefficients for uniformly loaded two-way element with two adjacent edges fixed and two edges simply supported.[8]

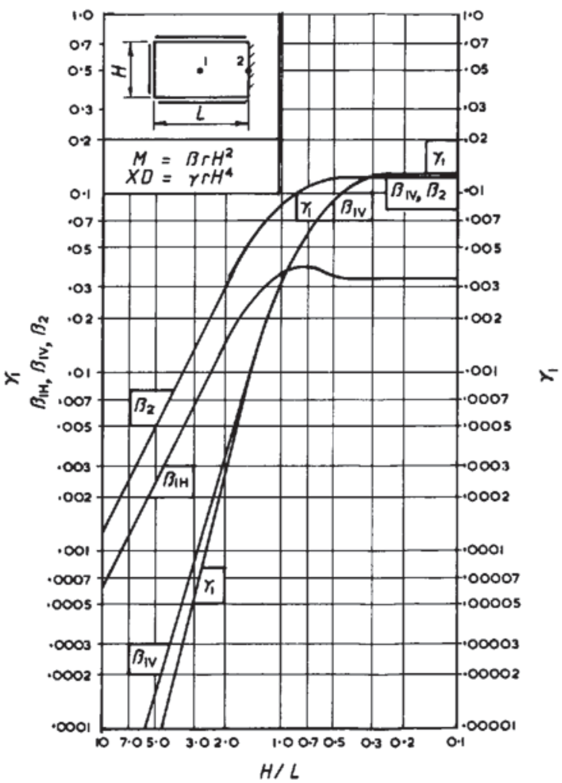


Fig. 3.17 Moment and deflection coefficients for uniformly loaded two-way element with three edges simply supported and one edge fixed.[8]

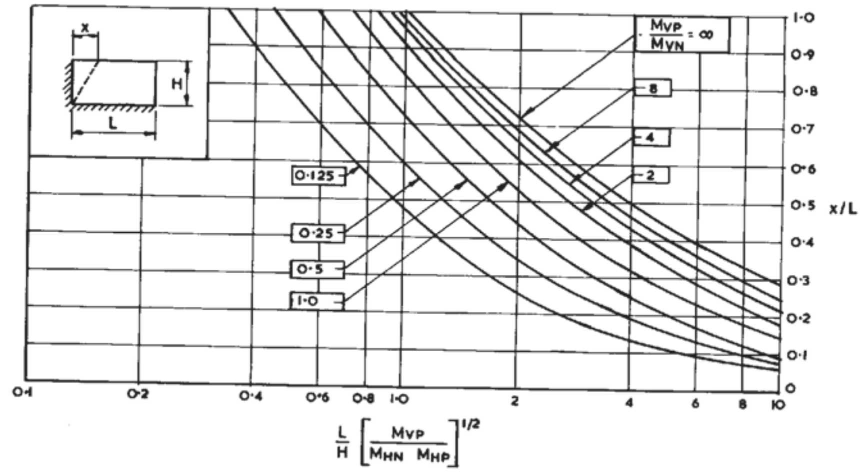


Fig. 3.18 Location of yield lines for two-way element with two adjacent edges supported and two edges free (values of x).[8]

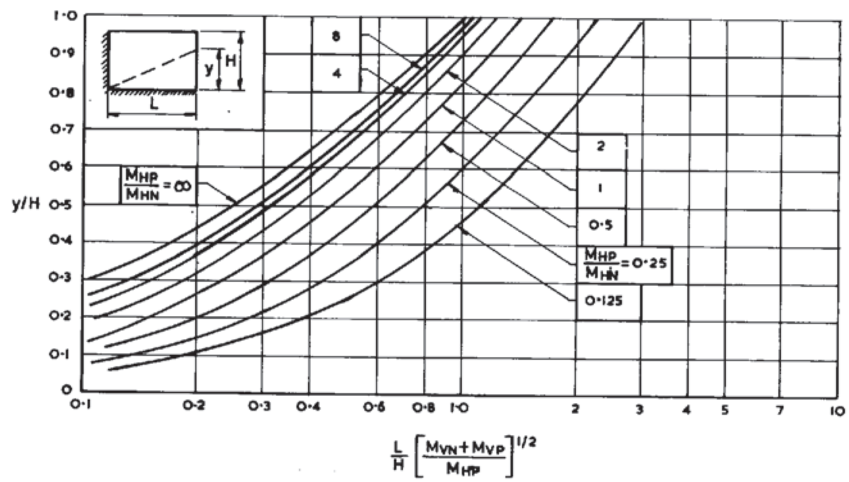


Fig. 3.19 Location of yield lines for two-way element with two adjacent edges supported and two edges free (values of y).[8]

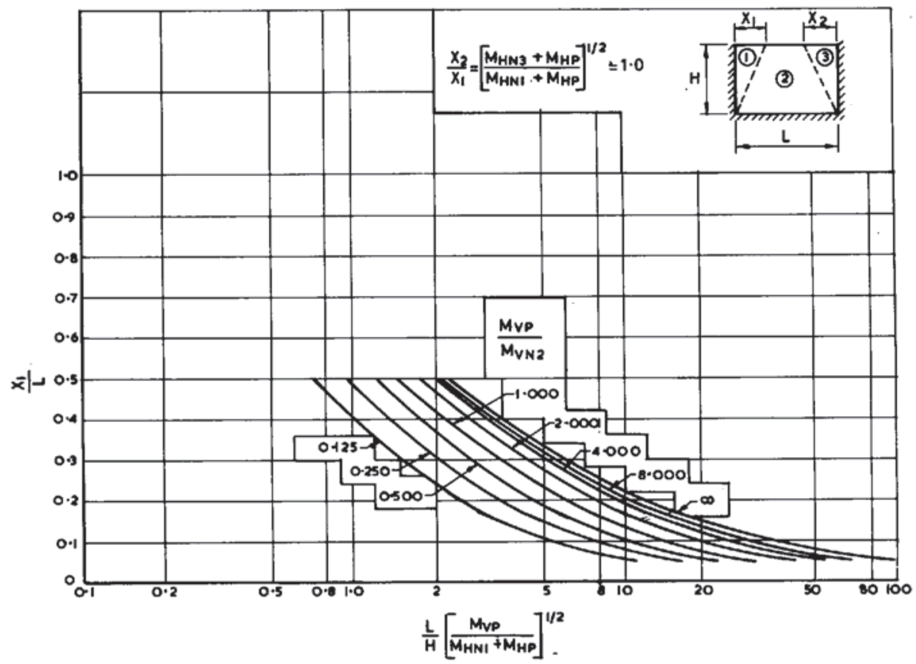


Fig. 3.20 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 1.0$).[8]

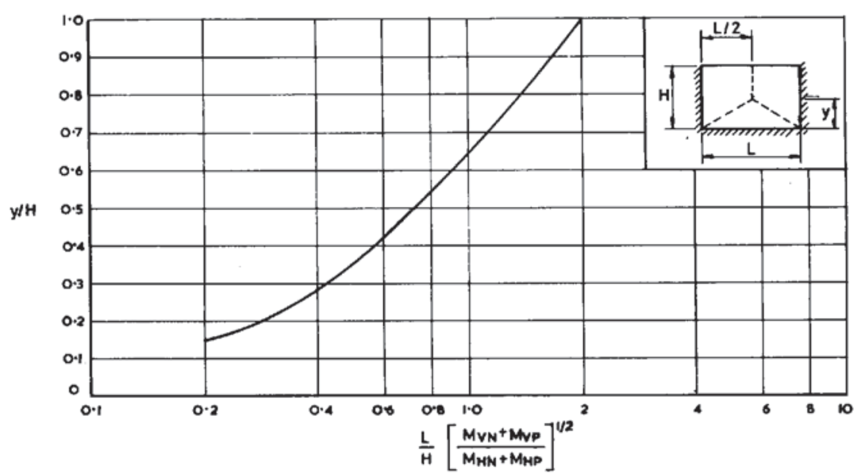


Fig. 3.21 Location of symmetrical yield lines for two-way element with three edges supported and one edge free (value of ν).[8]

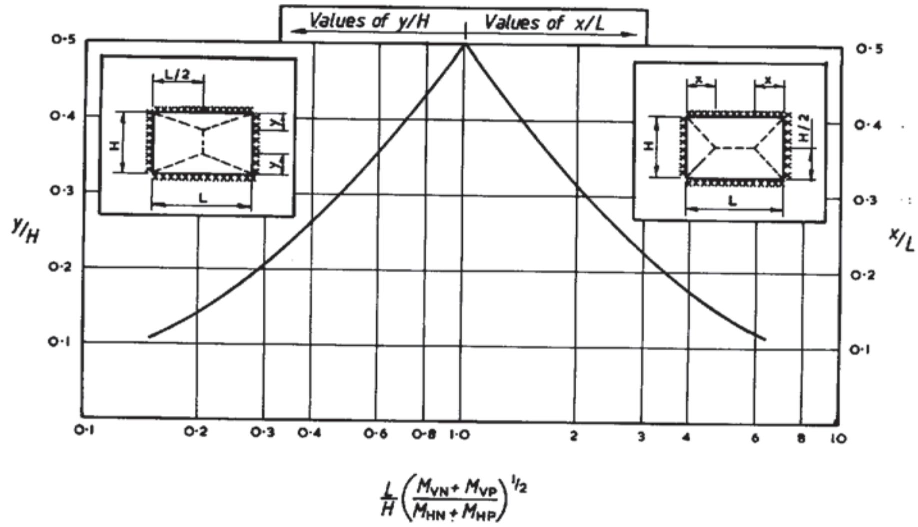


Fig. 3.22 Location of symmetrical yield lines for two-way element with four edges supported.[8]

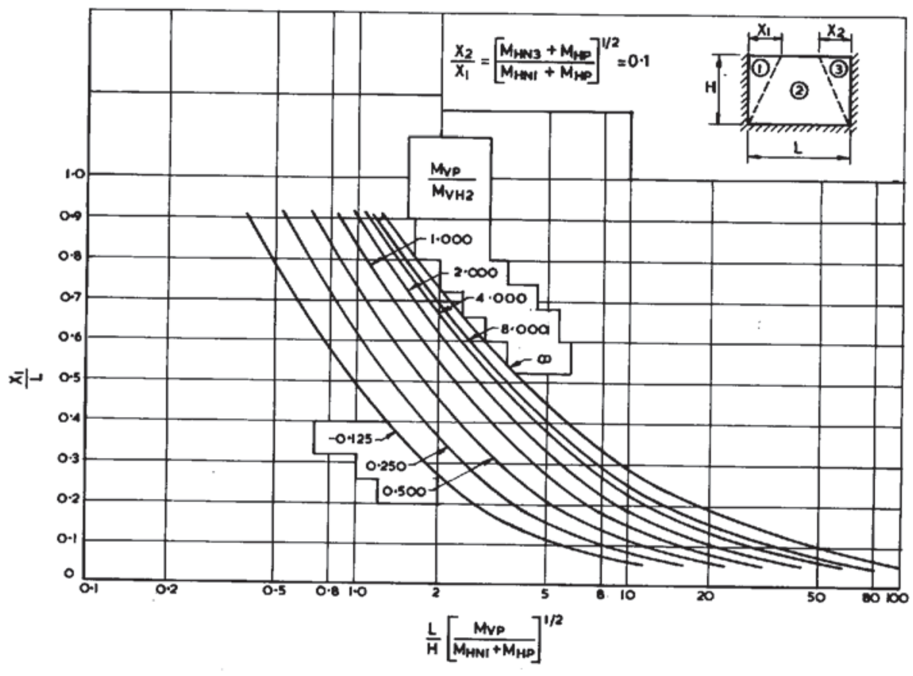


Fig. 3.23 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 0.1$).[8]

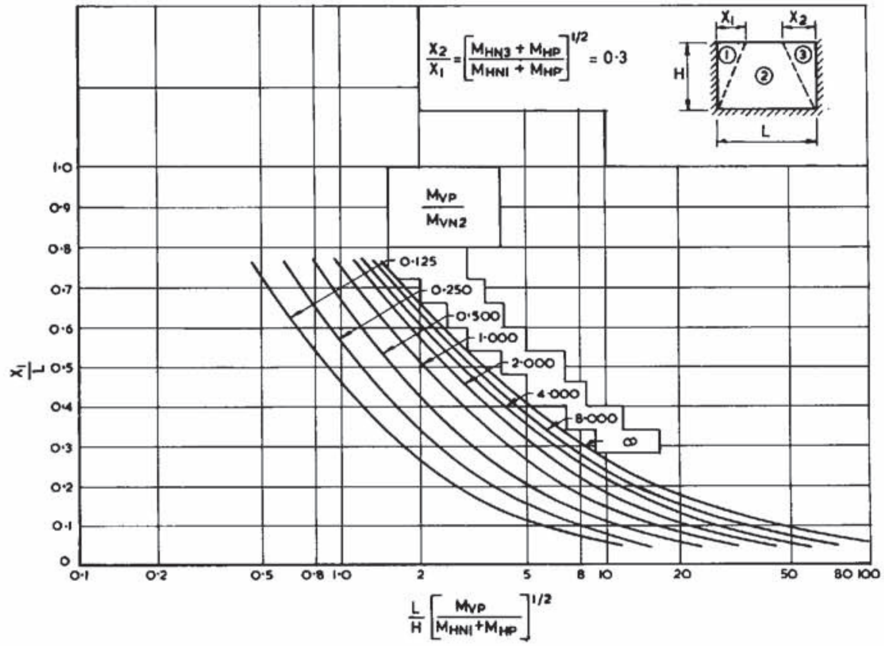


Fig. 3.24 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 0.3$).[8]

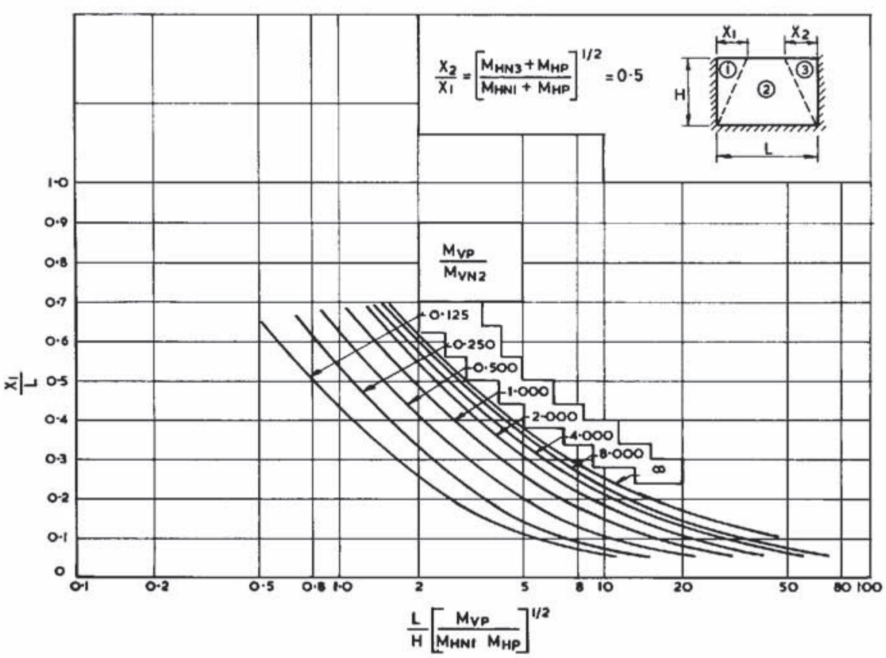


Fig. 3.25 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 0.5$).[8]

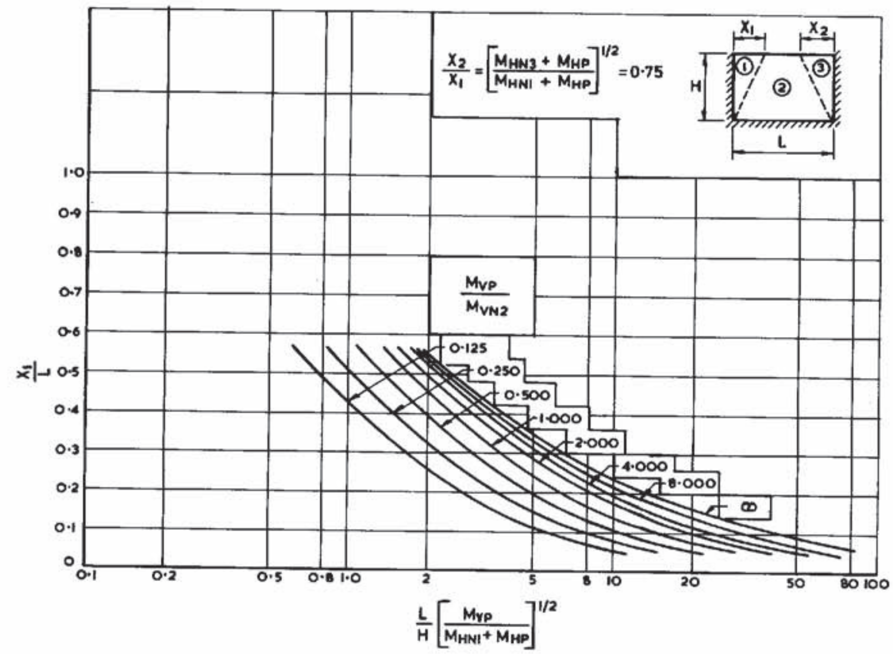


Fig. 3.26 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 0.75$).[8]

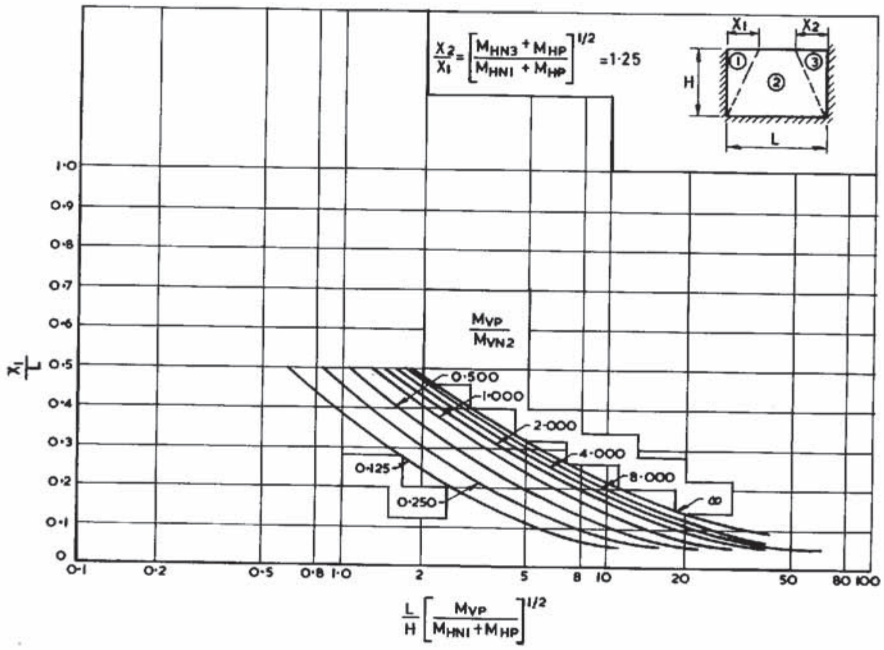


Fig. 3.27 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 1.25$).[8]

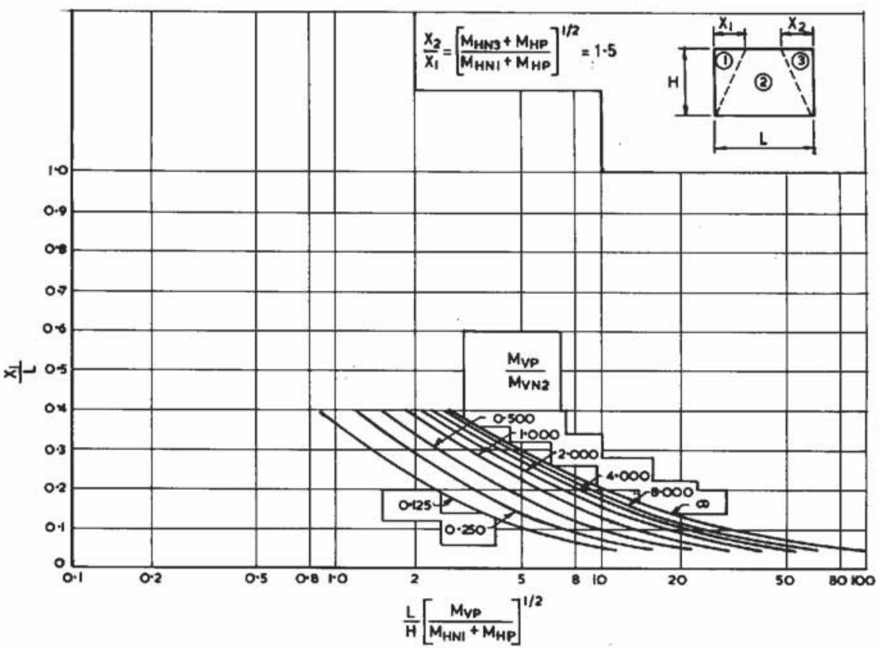


Fig. 3.28 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 1.5$).[8]

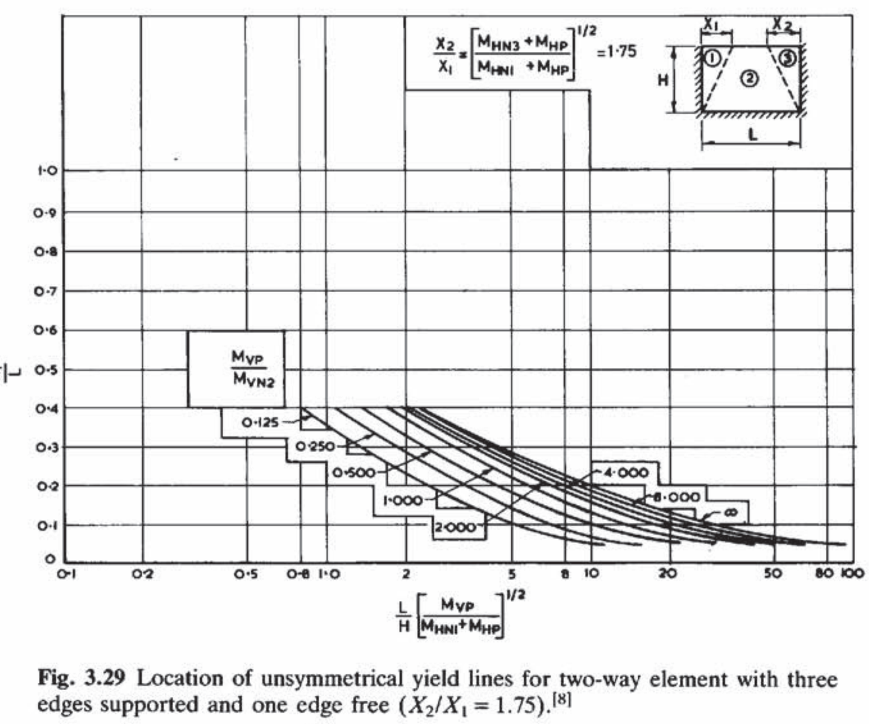


Fig. 3.29 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 1.75$).[8]

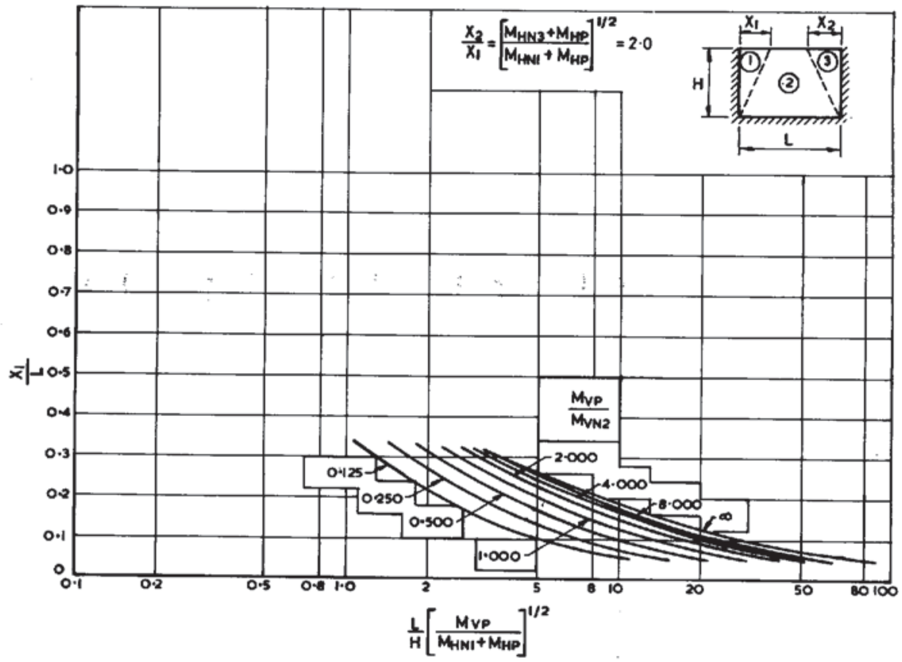


Fig. 3.30 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free ($X_2/X_1 = 2.0$).[8]

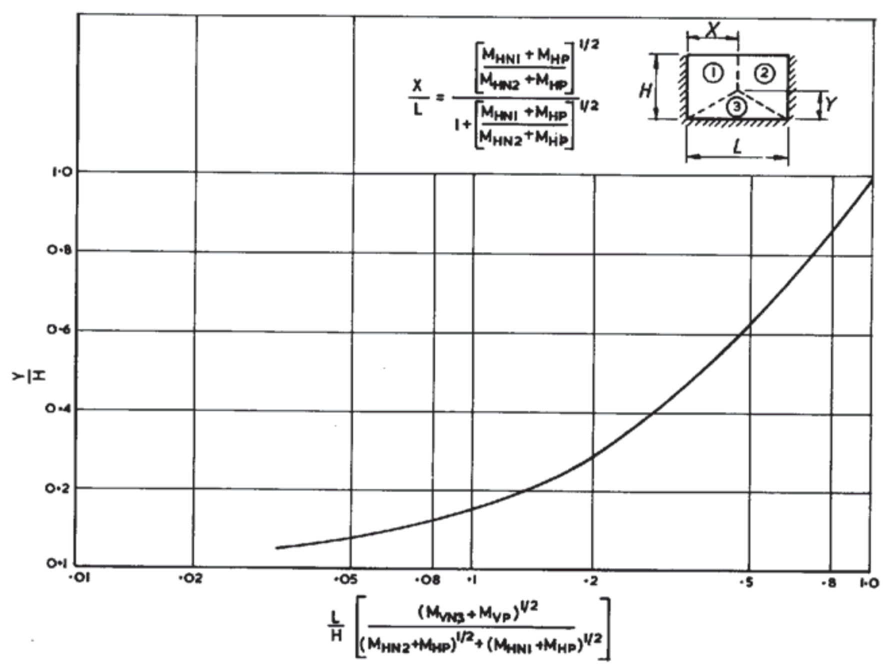


Fig. 3.31 Location of unsymmetrical yield lines for two-way element with three edges supported and one edge free (values of y).[8]

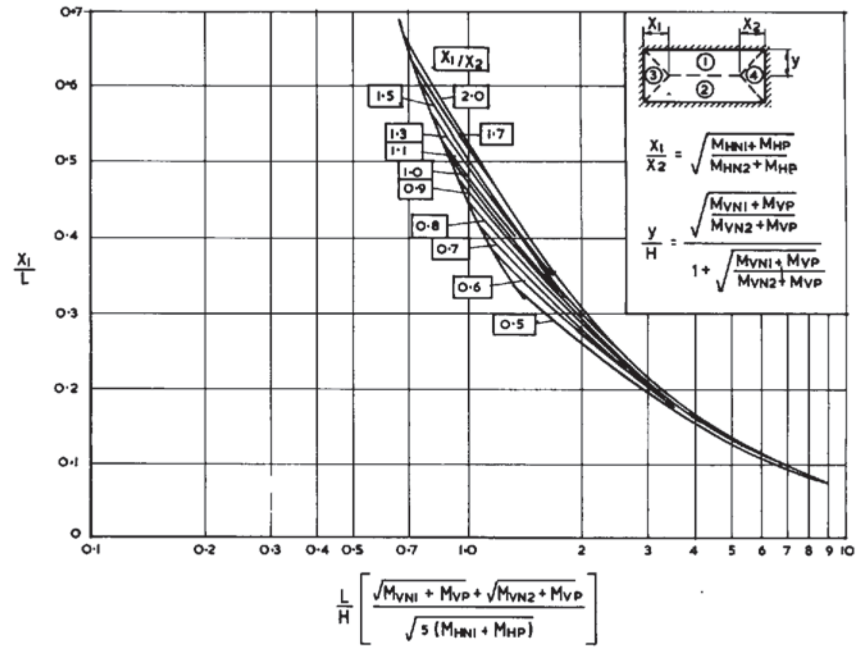


Fig. 3.32 Location of unsymmetrical yield lines for two-way element with four edges supported (values of X_1).[8]

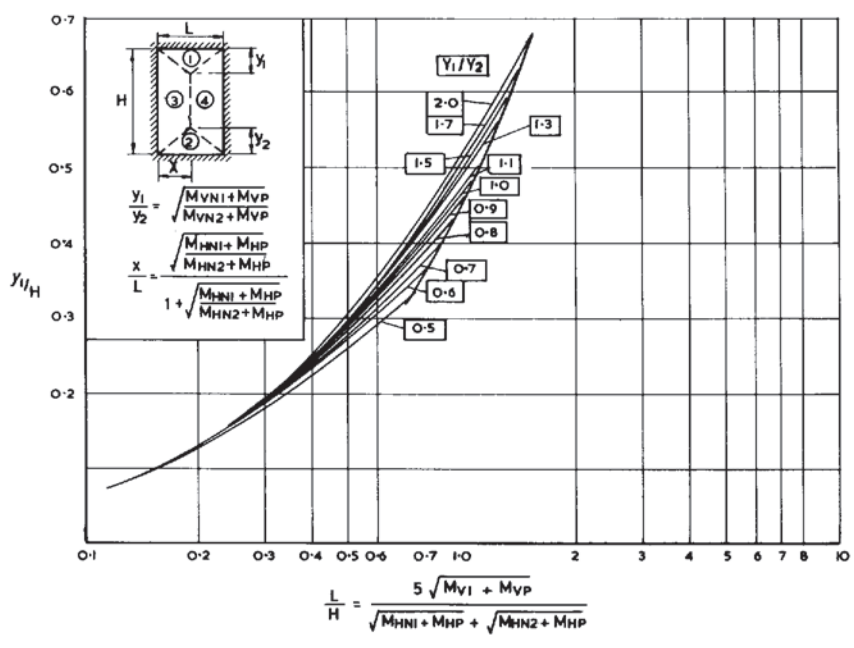


Fig. 3.33 Location of unsymmetrical yield lines for two-way element with four edges supported (values of Y_1).[8]

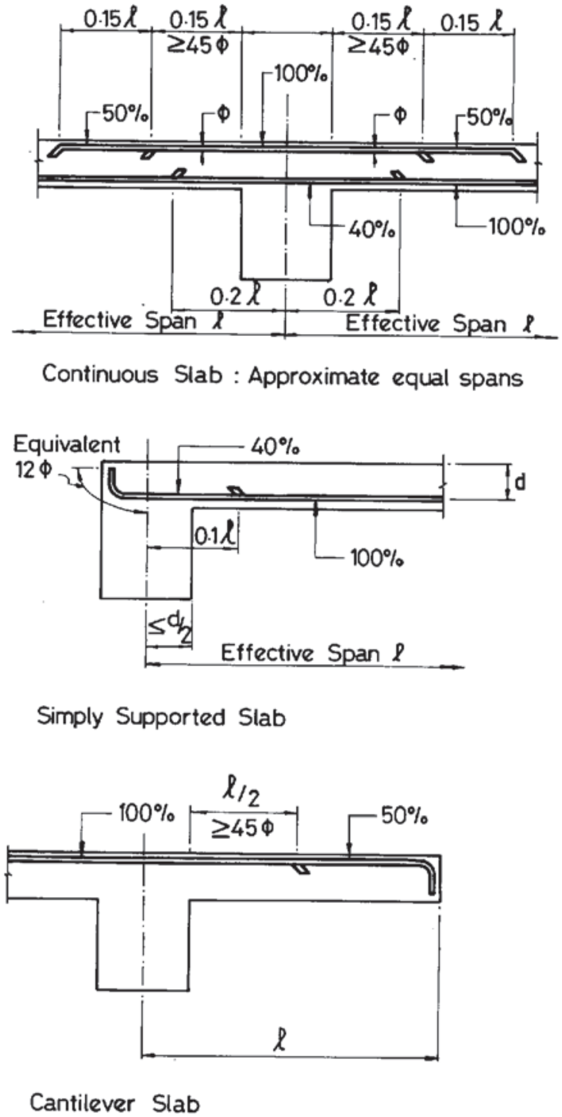


Fig. 3.34 Simplified detailing rules for slabs.

Table 3.1 Graphical summary of two-way elements to be used in conjunction with Figures 3.3 to 3.17

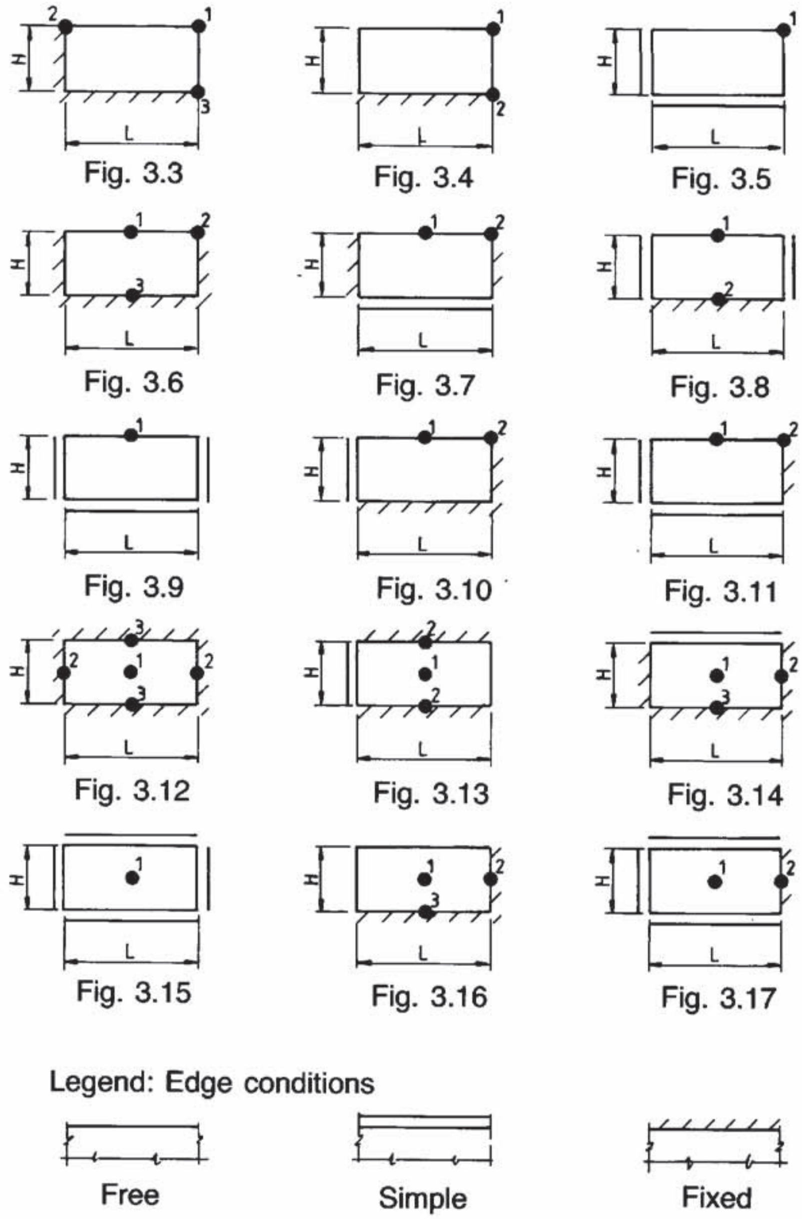


Table 3.2 Ultimate unit resistance for two-way elements (symmetrical yield-lines) (to be used in conjunction with Figs 3.18 to 3.23).

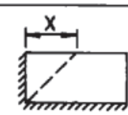
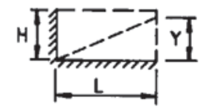
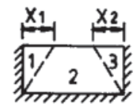
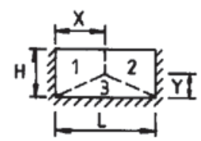
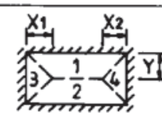
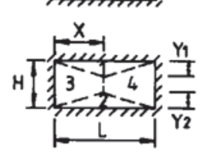
Edge conditions	Yield line locations	Limits	Ultimate unit resistance
Two adjacent edges supported and two edges free		$x \leq L$	$\dfrac{5(M_{HN} + M_{HP})}{x^2}$ or $\dfrac{6L\ M_{VN} + (5M_{VP} - M_{VN})x}{H^2(3L - 2x)}$
		$y \leq H$	$\dfrac{5(M_{VN} + M_{VP})}{y^2}$ or $\dfrac{6H\ M_{HN} + (5M_{HP} - M_{HN})y}{L^2(3H - 2y)}$
Three edges supported and one edge free		$x \leq \dfrac{L}{2}$	$\dfrac{5(M_{HN} + M_{HP})}{x^2}$ or $\dfrac{2M_{VN}(3L - x) + 10\ x\ M_{VP}}{H^2(3L - 4x)}$
		$y \leq H$	$\dfrac{5(M_{VN} + M_{VP})}{y^2}$ or $\dfrac{4(M_{HN} + M_{HP})(6H - y)}{L^2(3H - 2y)}$
Four edges supported		$x \leq \dfrac{L}{2}$	$\dfrac{5(M_{HN} + M_{HP})}{x^2}$ or $\dfrac{8(M_{VN} + M_{VP})(3L - x)}{H^2(3L - 4x)}$
		$y \leq \dfrac{H}{2}$	$\dfrac{5(M_{VN} + M_{VP})}{y^2}$ or $\dfrac{8(M_{HN} + M_{HP})(3H - y)}{L^2(3H - 4y)}$

Table 3.3 Ultimate unit resistance for two-way elements (unsymmetrical yield-lines) (to be used in conjunction with Figs 3.18 to 3.33).

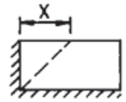
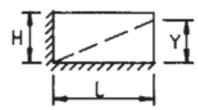
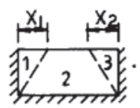
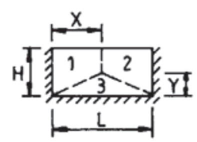
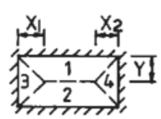
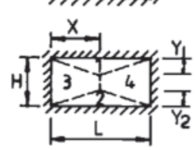
Edge conditions	Yield line locations	Limits	Ultimate unit resistance
Two adjacent edges supported and two edges free	 	$x \leq L$ $y \leq H$	Same as in Table 3.2
Three edges supported and one edge free	 	$x \leq \frac{L}{2}$ $y \leq H$	$\frac{5(M_{HN1} + M_{HP1})}{X_1^2}$ or $\frac{5(M_{HN3} + M_{HP})}{X_2^2}$ or $\frac{(5M_{VP} - M_{VN2})(X_1 + X_2) + 6M_{VN2}L}{H^2\,(3L - 2X_1 - 2X_2)}$ $\frac{(M_{HN1} + M_{HP})(6H - Y)}{X^2\,(3H - 2Y)}$ or $\frac{(M_{HN2} + M_{HP})(6H - Y)}{(L - X)^2(3H - 2Y)}$ or $\frac{5(M_{VN3} + M_{VP})}{Y^2}$
Four edges supported	 	$x \leq \frac{L}{2}$ $y \leq \frac{H}{2}$	$\frac{(M_{VN1} + M_{VP})(6L - X_1 - X_2)}{Y^2\,(3L - 2X_1 - 2X_2)}$ or $\frac{(M_{VN2} + M_{VP})(6L - X_1 - X_2)}{(H - Y)^2\,(3L - 2X_1 - 2X_2)}$ $\frac{5(M_{HN1} + M_{HP})}{X_1^2}$ or $\frac{5(M_{HN2} + M_{HP})}{X_2^2}$ $\frac{5(M_{VN1} + M_{VP})}{Y_1^2}$ or $\frac{5(M_{VN2} + M_{VP})}{Y_2^2}$ $\frac{(M_{HN1} + M_{HP})(6H - Y_1 - Y_2)}{X^2\,(3H - 2Y_1 - 2Y_2)}$ or $\frac{(M_{HN2} + M_{HP})(6H - Y_1 - Y_2)}{(L - X)^2\,(3H - 2Y_1 - 2Y_2)}$

Table 3.4 Ultimate support shears for two-way elements (symmetrical yield-lines) (to be used in conjunction with Table 3.2).

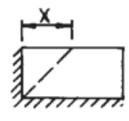
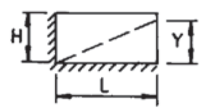
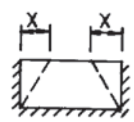
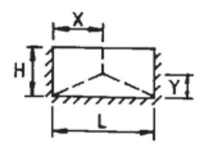
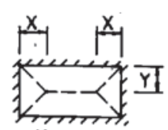
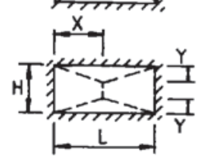
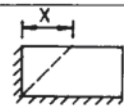
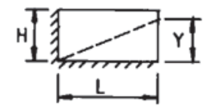
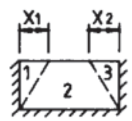
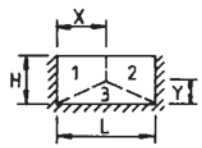
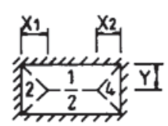
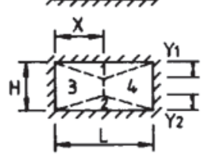
Edge conditions	Yield line locations	Limits	Horizontal shear, V_{sH}	Vertical shear, V_{sV}
Two adjacent edges supported and two edges free		$x \leq L$	$\dfrac{3r_u x}{5}$	$\dfrac{3r_u H\left(2 - \dfrac{x}{L}\right)}{\left(6 - \dfrac{x}{L}\right)}$
		$y \leq H$	$\dfrac{3r_u L\left(2 - \dfrac{y}{H}\right)}{\left(6 - \dfrac{y}{H}\right)}$	$\dfrac{3r_u y}{5}$
Three edges supported and one edge free		$x \leq \dfrac{L}{2}$	$\dfrac{3r_u x}{5}$	$\dfrac{3r_u H\left(1 - \dfrac{x}{L}\right)}{\left(3 - \dfrac{x}{L}\right)}$
		$y \leq H$	$\dfrac{3r_u L\left(2 - \dfrac{y}{H}\right)}{2\left(6 - \dfrac{y}{H}\right)}$	$\dfrac{3r_u y}{5}$
Four edges supported		$x \leq \dfrac{L}{2}$	$\dfrac{3r_u x}{5}$	$\dfrac{3r_u H\left(1 - \dfrac{x}{L}\right)}{2\left(3 - \dfrac{x}{L}\right)}$
		$y \leq \dfrac{H}{2}$	$\dfrac{3r_u L\left(1 - \dfrac{y}{H}\right)}{2\left(3 - \dfrac{y}{H}\right)}$	$\dfrac{3r_u y}{5}$

Table 3.5 Ultimate support shears for two-way elements (unsymmetrical yield-lines) (to be used in conjunction with Table 3.3).

Edge conditions	Yield line locations	limits	Horizontal shear, V_{sH}	Vertical shear, V_{sV}
Two adjacent edges supported and two edges free	 	$x \le L$ $y \le H$	Same as in Table 3.4	Same as in Table 3.4
Three edges supported and one edge free		$x_1 \le \frac{L}{2}$ $x_2 \le \frac{L}{2}$	$\frac{3x_1r_u}{5}$ $\frac{3x_2r_u}{5}$	$\frac{3r_uH(2L - x_1 - x_2)}{6L - x_1 - x_2}$
		$y \le H$	$\frac{3r_ux(2H - y)}{6H - y}$ $\frac{3r_ux\,(L - x)(2H - y)}{6H - y}$	$\frac{3r_uy}{5}$
Four edges supported		$x_1 \le \frac{L}{2}$ $x_2 \le \frac{L}{2}$	$\frac{3r_ux_1}{5}$ $\frac{3r_ux_2}{5}$	$\frac{3r_uy(2L - x_1 - x_2)}{6L - x_1 - x_2}$ $\frac{3r_u(H - y)(2L - x_1 - x_2)}{6L - x_1 - x_2}$
		$y_1 \le \frac{H}{2}$ $y_2 \le \frac{H}{2}$	$\frac{3r_ux\,(2H - y_1 - y_2)}{6H - y_1 - y_2}$ $\frac{3r_u(L - x)(2H - y_1 - y_2)}{6H - y_1 - y_2}$	$\frac{3r_uy_1}{5}$ $\frac{3r_uy_2}{5}$