

CHAPTER 3. COMPRESSION MEMBER DESIGN

3.1 INTRODUCTORY CONCEPTS

- Compression Members: Structural elements that are subjected to axial compressive forces only are called *columns*. Columns are subjected to axial loads thru the centroid.
- Stress: The stress in the column cross-section can be calculated as

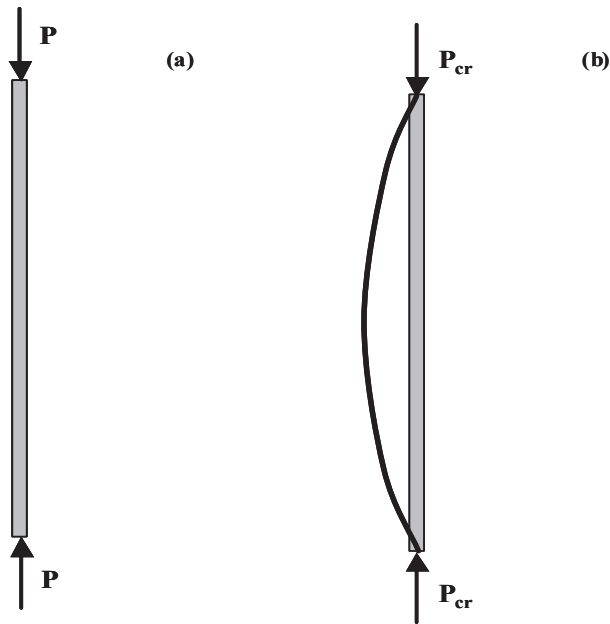
$$f = \frac{P}{A} \quad (2.1)$$

where, f is assumed to be uniform over the entire cross-section.

- This ideal state is never reached. The stress-state will be non-uniform due to:
 - Accidental eccentricity of loading with respect to the centroid
 - Member out-of-straightness (crookedness), or
 - Residual stresses in the member cross-section due to fabrication processes.
- Accidental eccentricity and member out-of-straightness can cause bending moments in the member. However, these are secondary and are usually ignored.
- Bending moments cannot be neglected if they are acting on the member. Members with axial compression and bending moment are called *beam-columns*.

3.2 COLUMN BUCKLING

- Consider a long slender compression member. If an axial load P is applied and increased slowly, it will ultimately reach a value P_{cr} that will cause buckling of the column. P_{cr} is called the critical buckling load of the column.



What is buckling?

Buckling occurs when a straight column subjected to axial compression suddenly undergoes bending as shown in the Figure 1(b). Buckling is identified as a failure limit-state for columns.

Figure 1. Buckling of axially loaded compression members

- The critical buckling load P_{cr} for columns is theoretically given by Equation (3.1)








$$P_{cr} = \frac{\pi^2 E I}{(K L)^2} \quad (3.1)$$

where, I = moment of inertia about axis of buckling

K = effective length factor based on end boundary conditions

- Effective length factors are given on page 16.1-189 of the AISC manual.

**Table C-C2.1
K Values for Columns**

<p>Buckled shape of column is shown by dashed line</p>						
<p>Theoretical K Value</p>	<p>0.5</p>	<p>0.7</p>	<p>1.0</p>	<p>1.0</p>	<p>2.0</p>	<p>2.0</p>
<p>Recommended design value when ideal condition are approximated</p>	<p>0.65</p>	<p>0.80</p>	<p>1.2</p>	<p>1.0</p>	<p>2.10</p>	<p>2.0</p>
<p>End Condition code</p>	 <p> <i>Rotation fixed and translation fixed</i> <i>Rotation free and translation fixed</i> <i>Rotation fixed and translation free</i> <i>Rotation free and translation free</i> </p>					

- In examples, homeworks, and exams please state clearly whether you are using the theoretical value of K or the recommended design values.

EXAMPLE 3.1 Determine the buckling strength of a W 12 x 50 column. Its length is 20 ft. For major axis buckling, it is pinned at both ends. For minor buckling, is it pinned at one end and fixed at the other end.

Solution

Step I. Visualize the problem

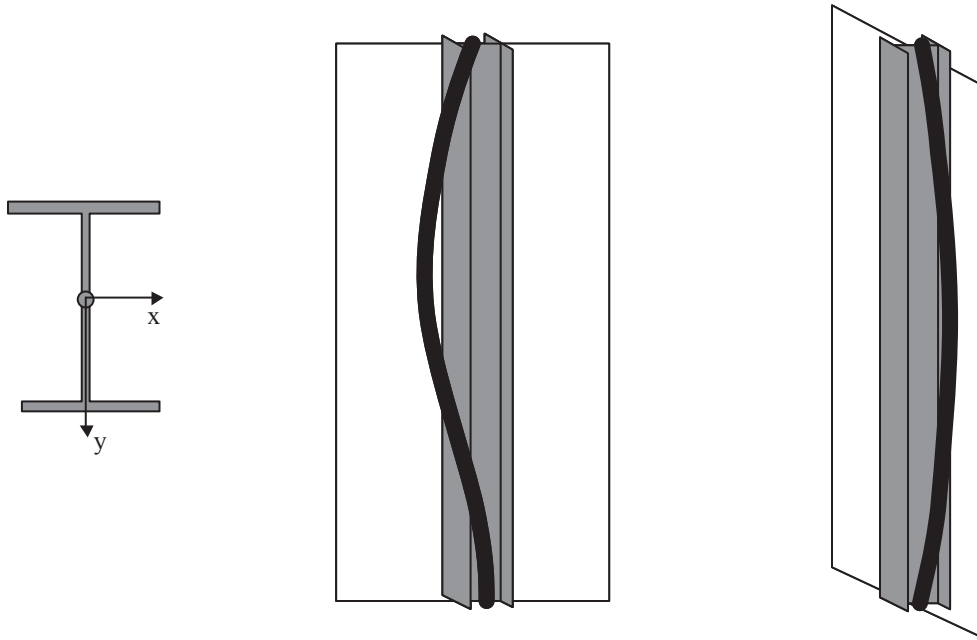


Figure 2. (a) Cross-section; (b) major-axis buckling; (c) minor-axis buckling

- For the W12 x 50 (or any wide flange section), x is the major axis and y is the minor axis.
Major axis means axis about which it has greater moment of inertia ($I_x > I_y$)

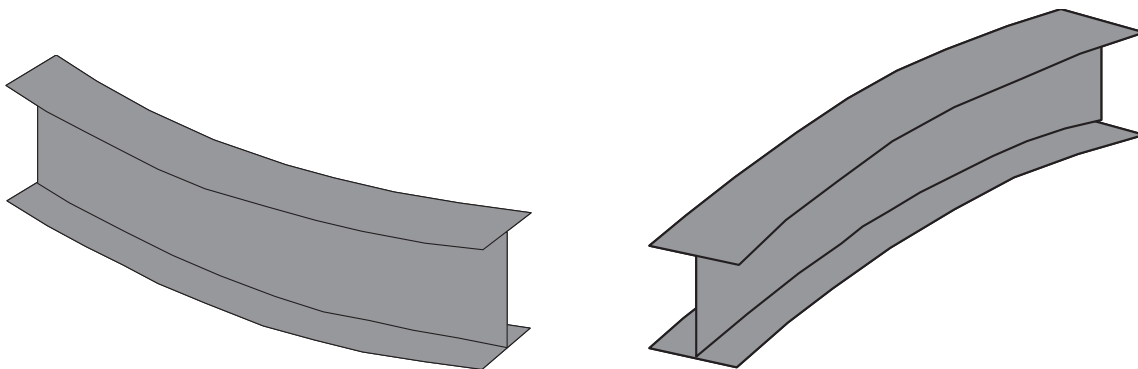


Figure 3. (a) Major axis buckling; (b) minor axis buckling

Step II. Determine the effective lengths

- According to Table C-C2.1 of the AISC Manual (see page 16.1 - 189):
 - For pin-pin end conditions about the minor axis
 $K_y = 1.0$ (theoretical value); and $K_y = 1.0$ (recommended design value)
 - For pin-fix end conditions about the major axis
 $K_x = 0.7$ (theoretical value); and $K_x = 0.8$ (recommended design value)
- According to the problem statement, the unsupported length for buckling about the major (x) axis = $L_x = 20$ ft.
- The unsupported length for buckling about the minor (y) axis = $L_y = 20$ ft.
- Effective length for major (x) axis buckling = $K_x L_x = 0.8 \times 20 = 16$ ft. = 192 in.
- Effective length for minor (y) axis buckling = $K_y L_y = 1.0 \times 20 = 20$ ft. = 240 in.

Step III. Determine the relevant section properties

- For W12 x 50: elastic modulus = $E = 29000$ ksi (constant for all steels)
- For W12 x 50: $I_x = 391$ in⁴. $I_y = 56.3$ in⁴ (see page 1-21 of the AISC manual)

Step IV. Calculate the buckling strength

- Critical load for buckling about x - axis = $P_{cr-x} = \frac{\pi^2 E I_x}{(K_x L_x)^2} = \frac{\pi^2 \times 29000 \times 391}{(192)^2}$

$$P_{cr-x} = 3035.8 \text{ kips}$$

- Critical load for buckling about y-axis = $P_{cr-y} = \frac{\pi^2 E I_y}{(K_y L_y)^2} = \frac{\pi^2 \times 29000 \times 56.3}{(240)^2}$

$$P_{cr-y} = 279.8 \text{ kips}$$

- Buckling strength of the column = smaller (P_{cr-x}, P_{cr-y}) = $P_{cr} = 279.8$ kips

Minor (y) axis buckling governs.

- **Notes:**

- *Minor axis buckling usually governs for all doubly symmetric cross-sections. However, for some cases, major (x) axis buckling can govern.*
- *Note that the steel yield stress was irrelevant for calculating this buckling strength.*

3.3 INELASTIC COLUMN BUCKLING

- Let us consider the previous example. According to our calculations $P_{cr} = 279.8$ kips. This P_{cr} will cause a uniform stress $f = P_{cr}/A$ in the cross-section
- For W12 x 50, $A = 14.6 \text{ in}^2$. Therefore, for $P_{cr} = 279.8$ kips; $f = 19.16$ ksi
The calculated value of f is within the elastic range for a 50 ksi yield stress material.
- However, if the unsupported length was only 10 ft., $P_{cr} = \frac{\pi^2 E I_y}{(K_y L_y)^2}$ would be calculated as 1119 kips, and $f = 76.6$ kips.
- This value of f is ridiculous because the material will yield at 50 ksi and never develop $f = 76.6$ kips. The member would yield before buckling.
- **Equation (3.1) is valid only when the material everywhere in the cross-section is in the elastic region. If the material goes inelastic then Equation (3.1) becomes useless and cannot be used.**
- What happens in the inelastic range?
Several other problems appear in the inelastic range.
 - The member out-of-straightness has a significant influence on the buckling strength in the inelastic region. It must be accounted for.

- The residual stresses in the member due to the fabrication process causes yielding in the cross-section much before the uniform stress f reaches the yield stress F_y .
- The shape of the cross-section (W, C, etc.) also influences the buckling strength.
- In the inelastic range, the steel material can undergo strain hardening.

All of these are very advanced concepts and beyond the scope of CE405. You are welcome to CE805 to develop a better understanding of these issues.

- So, what should we do? We will directly look at the AISC Specifications for the strength of compression members, i.e., Chapter E (page 16.1-27 of the AISC manual).

3.4 AISC SPECIFICATIONS FOR COLUMN STRENGTH

- The AISC specifications for column design are based on several years of research.
- These specifications account for the elastic and inelastic buckling of columns including all issues (member crookedness, residual stresses, accidental eccentricity etc.) mentioned above.
- The specification presented here (AISC Spec E2) will work for all doubly symmetric cross-sections and channel sections.
- The design strength of columns for the flexural buckling limit state is equal to $\phi_c P_n$

Where, $\phi_c = 0.85$ (Resistance factor for compression members)

$$P_n = A_g F_{cr} \quad (3.2)$$

- For $\lambda_c \leq 1.5$ $F_{cr} = (0.658^{\lambda_c^2}) F_y$ (3.3)

- For $\lambda_c > 1.5$ $F_{cr} = \left[\frac{0.877}{\lambda_c^2} \right] F_y$ (3.4)

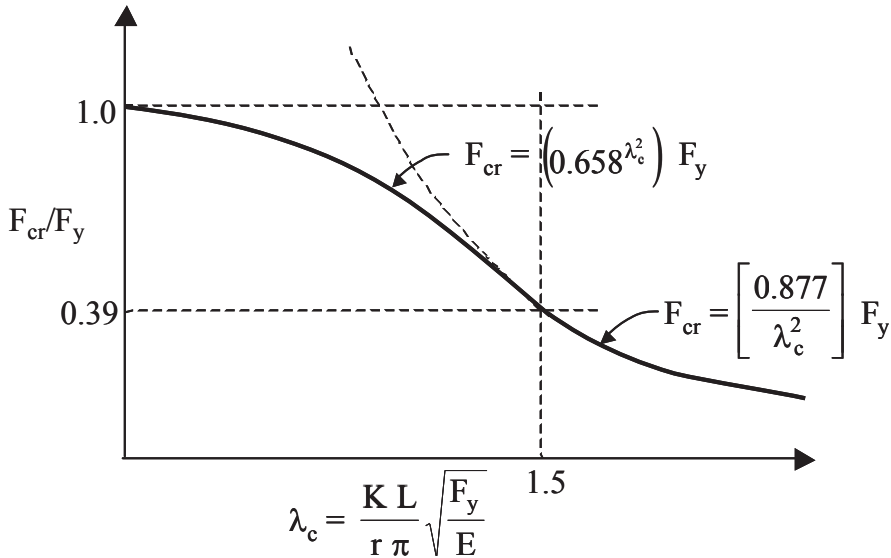
Where, $\lambda_c = \frac{K L}{r \pi} \sqrt{\frac{F_y}{E}}$ (3.5)

A_g = gross member area;

K = effective length factor

L = unbraced length of the member;

r = governing radius of gyration



- Note that the original Euler buckling equation is $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

$$\therefore F_{cr} = \frac{P_{cr}}{A_g} = \frac{\pi^2 E}{(KL)^2} \times \frac{I}{A_g} = \frac{\pi^2 E}{(KL)^2} \times r^2 = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

$$\therefore \frac{F_{cr}}{F_y} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2 \times F_y} = \frac{1}{\left(\frac{KL}{r \pi} \times \sqrt{\frac{F_y}{E}}\right)^2} = \frac{1}{\lambda_c^2}$$

$$\therefore F_{cr} = F_y \times \frac{1}{\lambda_c^2}$$

- Note that the AISC equation for $\lambda_c < 1.5$ is $F_{cr} = F_y \times \frac{0.877}{\lambda_c^2}$

- The 0.877 factor tries to account for initial crookedness.

- For a given column section:

- Calculate I , A_g , r

- Determine effective length KL based on end boundary conditions.

- Calculate λ_c

- If λ_c is greater than 1.5, *elastic buckling* occurs and use Equation (3.4)

- If λ_c is less than or equal to 1.5, *inelastic buckling* occurs and use Equation (3.3)
- Note that the column can develop its yield strength F_y as λ_c approaches zero.

3.5 COLUMN STRENGTH

- In order to simplify calculations, the AISC specification includes Tables.
 - Table 3-36 on page 16.1-143 shows KL/r vs. $\phi_c F_{cr}$ for steels with $F_y = 36$ ksi.
 - You can calculate KL/r for the column, then read the value of $\phi_c F_{cr}$ from this table
 - The column strength will be equal to $\phi_c F_{cr} \times A_g$
 - Table 3-50 on page 16.1-145 shows KL/r vs. $\phi_c F_{cr}$ for steels with $F_y = 50$ ksi.
- In order to simplify calculations, the AISC specification includes more Tables.
 - Table 4 on page 16.1-147 shows λ_c vs. $\phi_c F_{cr}/F_y$ for all steels with any F_y .
 - You can calculate λ_c for the column, then read the value of $\phi_c F_{cr}/F_y$
 - The column strength will be equal to $\phi_c F_{cr}/F_y \times (A_g \times F_y)$

EXAMPLE 3.2 Calculate the design strength of W14 x 74 with length of 20 ft. and pinned ends.
A36 steel is used.

Solution

- Step I. Calculate the effective length and slenderness ratio for the problem

$$K_x = K_y = 1.0$$

$$L_x = L_y = 240 \text{ in.}$$

$$\text{Major axis slenderness ratio} = K_x L_x / r_x = 240 / 6.04 = 39.735$$

$$\text{Minor axis slenderness ratio} = K_y L_y / r_y = 240 / 2.48 = 96.77$$

- Step II. Calculate the buckling strength for governing slenderness ratio

The governing slenderness ratio is the larger of ($K_x L_x / r_x$, $K_y L_y / r_y$)

$K_y L_y / r_y$ is larger and the governing slenderness ratio; $\lambda_c = \frac{K_y L_y}{r_y \pi} \sqrt{\frac{F_y}{E}} = 1.085$

$\lambda_c < 1.5$; Therefore, $F_{cr} = (0.658^{\lambda_c^2}) F_y$

Therefore, $F_{cr} = 21.99$ ksi

Design column strength = $\phi_c P_n = 0.85 (A_g F_{cr}) = 0.85 (21.8 \text{ in}^2 \times 21.99 \text{ ksi}) = 408$ kips

Design strength of column = 408 kips

- Check calculated values with Table 3-36. For $KL/r = 97$, $\phi_c F_{cr} = 18.7$ ksi
- Check calculated values with Table 4. For $\lambda_c = 1.08$, $\phi_c F_{cr} = 0.521$

3.6 LOCAL BUCKLING LIMIT STATE

- The AISC specifications for column strength assume that column buckling is the governing limit state. However, if the column section is made of thin (slender) plate elements, then failure can occur due to *local buckling* of the flanges or the webs.

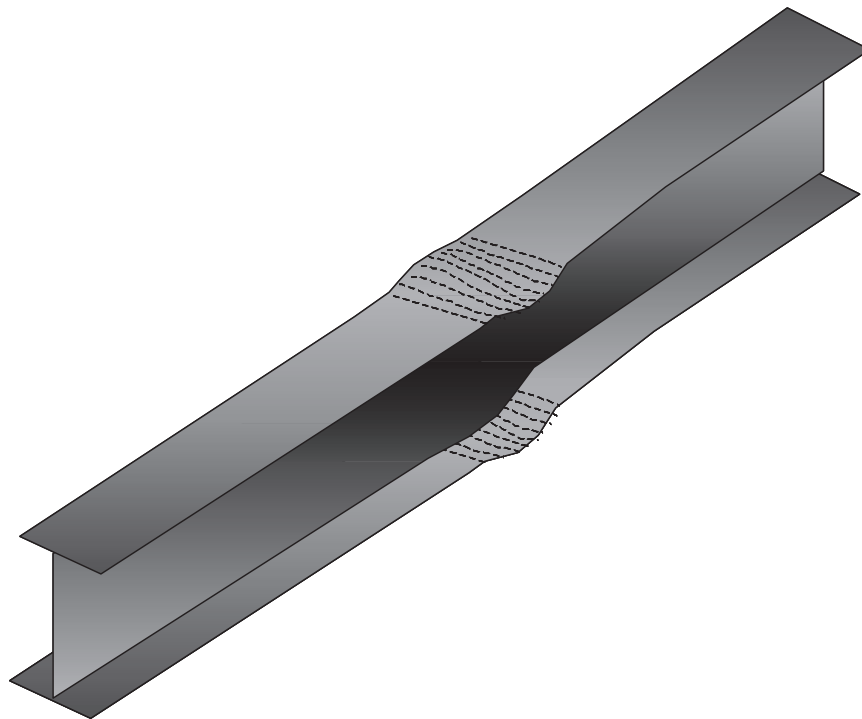


Figure 4. Local buckling of columns

- If *local buckling* of the individual plate elements occurs, then the column may not be able to develop its buckling strength.
- Therefore, the local buckling limit state must be prevented from controlling the column strength.
- Local buckling depends on the slenderness (width-to-thickness b/t ratio) of the plate element and the yield stress (F_y) of the material.
- Each plate element must be stocky enough, i.e., have a b/t ratio that prevents local buckling from governing the column strength.

- The AISC specification B5 provides the slenderness (b/t) limits that the individual plate elements must satisfy so that *local buckling* does not control.
- The AISC specification provides two slenderness limits (λ_p and λ_r) for the local buckling of plate elements.

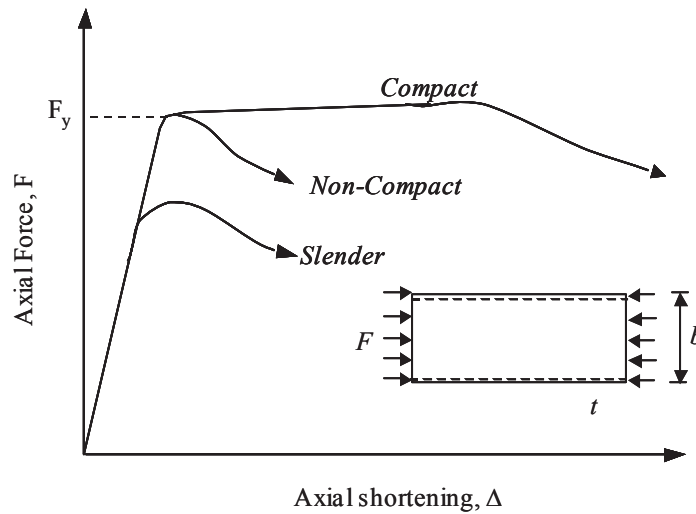


Figure 5. Local buckling behavior and classification of plate elements

- If the slenderness ratio (b/t) of the plate element is greater than λ_r then it is *slender*. It will locally buckle in the elastic range *before* reaching F_y
- If the slenderness ratio (b/t) of the plate element is less than λ_r but greater than λ_p , then it is *non-compact*. It will locally buckle *immediately* after reaching F_y
- If the slenderness ratio (b/t) of the plate element is less than λ_p , then the element is *compact*. It will locally buckle *much* after reaching F_y
- If all the plate elements of a cross-section are compact, then the section is *compact*.
 - If any one plate element is non-compact, then the cross-section is non-compact
 - If any one plate element is slender, then the cross-section is slender.
- The slenderness limits λ_p and λ_r for various plate elements with different boundary conditions are given in Table B5.1 on pages 16.1-14 and 16.1-15 of the AISC Spec.

- Note that the slenderness limits (λ_p and λ_r) and the definition of plate slenderness (b/t) ratio depend upon the boundary conditions for the plate.
 - If the plate is supported along *two edges* parallel to the direction of compression force, then it is a *stiffened* element. For example, the webs of W shapes
 - If the plate is supported along only *one edge* parallel to the direction of the compression force, then it is an *unstiffened* element. Ex., the flanges of W shapes.
- The local buckling limit state can be prevented from controlling the column strength by using sections that are non-compact
 - If all the elements of the cross-section have calculated slenderness (b/t) ratio less than λ_r , then the local buckling limit state will not control.
 - For the definitions of b/t, λ_p , λ_r for various situations see Table B5.1 and Spec B5.

EXAMPLE 3.3 Determine the local buckling slenderness limits and evaluate the W14 x 74 section used in Example 3.2. Does local buckling limit the column strength?

Solution

- Step I. Calculate the slenderness limits

See Table B5.1 on page 16.1 – 14.

- For the flanges of I-shape sections in pure compression

$$\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 0.56 \times \sqrt{\frac{29000}{36}} = 15.9$$

- For the webs of I-shapes section in pure compression

$$\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 0.56 \times \sqrt{\frac{29000}{36}} = 15.9$$

$$\lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 1.49 \times \sqrt{\frac{29000}{36}} = 42.3$$

- Step II. Calculate the slenderness ratios for the flanges and webs of W14 x 74

- For the flanges of I-shape member, $b = b_f/2 = \text{flange width} / 2$

Therefore, $b/t = b_f/2t_f$. (See pg. 16.1-12 of AISC)

For W 14 x 74, $b_f/2t_f = 6.41$ (See Page 1-19 in AISC)

- For the webs of I shaped member, $b = h$

h is the clear distance between flanges less the fillet / corner radius of each flange

For W14 x 74, $h/t_w = 25.4$ (See Page 1-19 in AISC)

- Step III. Make the comparisons and comment

For the flanges, $b/t < \lambda_r$. Therefore, the flange is non-compact

For the webs, $h/t_w < \lambda_r$. Therefore the web is non-compact

Therefore, the section is compact

Therefore, local buckling will not limit the column strength.

3.7 COLUMN DESIGN

- The AISC manual has tables for column strength. See page 4-21 onwards.
- For wide flange sections, *the column buckling strength ($\phi_c P_n$) is tabulated with respect to the effective length about the minor axis $K_y L_y$ in Table 4-2.*

- The table takes the $K_y L_y$ value for a section, and internally calculates the $K_y L_y / r_y$, then λ_c

$$= \frac{K_y L_y}{r_y \pi} \sqrt{\frac{F_y}{E}} ; \text{ and then the } \textit{tabulated} \text{ column strength using either Equation E2-2 or}$$

E2-3 of the specification.

- If you want to use the Table 4-2 for calculating the column strength for buckling about *the major axis*, then do the following:
 - Take the major axis $K_x L_x$ value. Calculate an equivalent $(KL)_{eq} = \frac{K_x L_x}{r_x / r_y}$
 - Use the calculated $(KL)_{eq}$ value to find $(\phi_c P_n)$ the column strength for buckling about the *major axis* from Table (4-2)
- For example, consider a W14 x 74 column with $K_y L_y = 20$ ft. and $K_x L_x = 25$ ft.
 - Material has yield stress = 50 ksi (always in Table 4-2).
 - See Table 4-2, for $K_y L_y = 20$ ft., $\phi_c P_n = 467$ kips (minor axis buckling strength)
 - r_x/r_y for W14x74 = 2.44 from Table 4-2 (see page 4-23 of AISC).
 - For $K_x L_x = 25$ ft., $(KL)_{eq} = 25/2.44 = 10.25$ ft.
 - For $(KL)_{eq} = 10.25$ ft., $\phi_c P_n = 774$ kips (major axis buckling strength)
 - If calculated value of $(KL)_{eq} < K_y L_y$ then minor axis buckling will govern.

EXAMPLE 3.4 Determine the design strength of an ASTM A992 W14 x 132 that is part of a braced frame. Assume that the physical length $L = 30$ ft., the ends are pinned and the column is braced at the ends only for the X-X axis and braced at the ends and mid-height for the Y-Y axis.

Solution

- Step I. Calculate the *effective lengths*.

For W14 x 132: $r_x = 6.28$ in; $r_y = 3.76$ in; $A_g = 38.8$ in²

$K_x = 1.0$ and $K_y = 1.0$

$L_x = 30$ ft. and $L_y = 15$ ft.

$K_x L_x = 30$ ft. and $K_y L_y = 15$ ft.

- Step II. Determine the governing slenderness ratio

$$K_x L_x / r_x = 30 \times 12 \text{ in.} / 6.28 \text{ in.} = 57.32$$

$$K_y L_y / r_y = 15 \times 12 \text{ in.} / 3.76 \text{ in.} = 47.87$$

The larger slenderness ratio, therefore, buckling about the major axis will govern the column strength.

- Step III. Calculate the column strength

$$K_x L_x = 30 \text{ ft.} \quad \text{Therefore, } (KL)_{eq} = \frac{K_x L_x}{r_x / r_y} = \frac{30}{6.28 / 3.76} = 17.96 \text{ ft.}$$

From Table 4-2, for $(KL)_{eq} = 18.0 \text{ ft.}$ $\phi_c P_n = 1300 \text{ kips}$ (design column strength)

- Step IV. Check the local buckling limits

$$\text{For the flanges, } b_f / 2t_f = 7.15 < \lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$$

$$\text{For the web, } h / t_w = 17.7 < \lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$$

Therefore, the section is non-compact. OK.

EXAMPLE 3.5 A compression member is subjected to service loads of 165 kips dead load and 535 kips of live load. The member is 26 ft. long and pinned at each end. Use A992 (50 ksi) steel and select a W shape

Solution

- Calculate the factored design load P_u

$$P_u = 1.2 P_D + 1.6 P_L = 1.2 \times 165 + 1.6 \times 535 = 1054 \text{ kips}$$

- Select a W shape from the AISC manual Tables

For $K_y L_y = 26 \text{ ft.}$ and required strength = 1054 kips

- Select W14 x 145 from page 4-22. It has $\phi_c P_n = 1160 \text{ kips}$

- Select W12 x 170 from page 4-24. It has $\phi_c P_n = 1070$ kips
- No no W10 will work. See Page 4-26
- W14 x 145 is the lightest.
- Note that column sections are usually W12 or W14. Usually sections bigger than W14 are usually not used as columns.

3.8 EFFECTIVE LENGTH OF COLUMNS IN FRAMES

- So far, we have looked at the buckling strength of individual columns. These columns had various boundary conditions at the ends, but they were not connected to other members with moment (fix) connections.
- The effective length factor K for the buckling of an individual column can be obtained for the appropriate end conditions from Table C-C2.1 of the AISC Manual .
- However, when these individual columns are part of a frame, their ends are connected to other members (beams etc.).
 - Their effective length factor K will depend on the restraint offered by the other members connected at the ends.
 - Therefore, the effective length factor K will depend on the relative rigidity (stiffness) of the members connected at the ends.

The effective length factor for columns in frames must be calculated as follows:

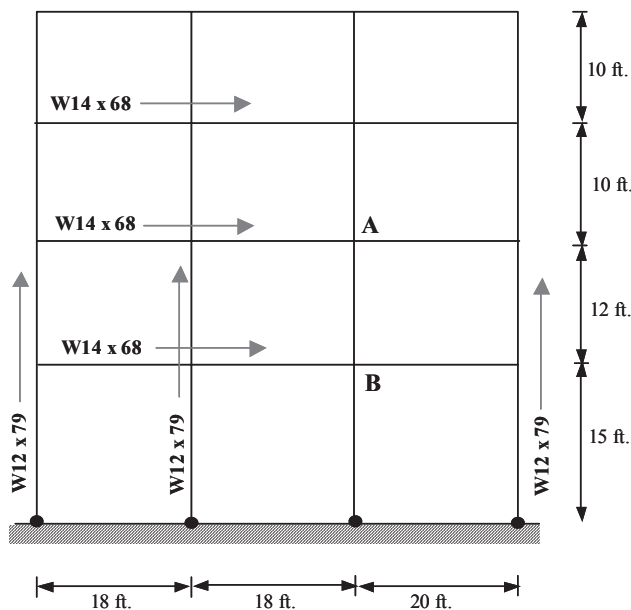
- First, you have to determine whether the column is part of a braced frame or an unbraced (moment resisting) frame.
 - If the column is part of a braced frame then its effective length factor $0 < K \leq 1$
 - If the column is part of an unbraced frame then $1 < K \leq \infty$

- Then, you have to determine the relative rigidity factor G for both ends of the column
 - G is defined as the ratio of the summation of the rigidity (EI/L) of all columns coming together at an end to the summation of the rigidity (EI/L) of all beams coming together at the same end.

- $G = \frac{\sum \frac{E I_c}{L_c}}{\sum \frac{E I_b}{L_b}}$ - It must be calculated for both ends of the column.

- Then, you can determine the effective length factor K for the column using the calculated value of G at both ends, i.e., G_A and G_B and the appropriate alignment chart
- There are two alignment charts provided by the AISC manual,
 - One is for columns in braced (sidesway inhibited) frames. See Figure C-C2.2a on page 16.1-191 of the AISC manual. $0 < K \leq 1$
 - The second is for columns in unbraced (sidesway uninhibited) frames. See Figure C-C2.2b on page 16.1-192 of the AISC manual. $1 < K \leq \infty$
 - The procedure for calculating G is the same for both cases.

EXAMPLE 3.6 Calculate the effective length factor for the **W12 x 53** column AB of the frame shown below. Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame. Assume that the columns are braced at each story level for out-of-plane buckling. Assume that the same column section is used for the stories above and below.



Step I. Identify the frame type and calculate L_x , L_y , K_x , and K_y if possible.

- It is an unbraced (sidesway uninhibited) frame.
- $L_x = L_y = 12$ ft.
- $K_y = 1.0$
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.

Step II - Calculate K_x

- I_{xx} of W 12 x 53 = 425 in⁴ I_{xx} of W14x68 = 753

$$\bullet \quad G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{10 \times 12} + \frac{425}{12 \times 12}}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{6.493}{6.360} = 1.021$$

$$\bullet \quad G_B = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{12 \times 12} + \frac{425}{15 \times 12}}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{5.3125}{6.360} = 0.835$$

- Using G_A and G_B : $K_x = 1.3$ - from Alignment Chart on Page 3-6

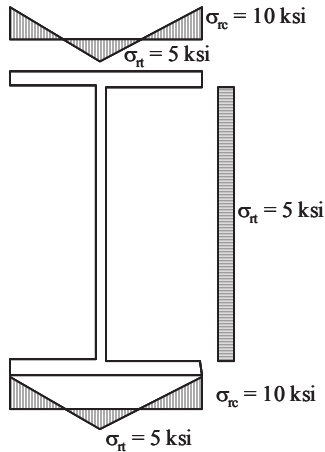
Step III – Design strength of the column

- $K_y L_y = 1.0 \times 12 = 12$ ft.
- $K_x L_x = 1.3 \times 12 = 15.6$ ft.
 - r_x / r_y for W12x53 = 2.11
 - $(KL)_{eq} = 15.6 / 2.11 = 7.4$ ft.
- $K_y L_y > (KL)_{eq}$
- Therefore, y-axis buckling governs. Therefore $\phi_c P_n = 518$ kips

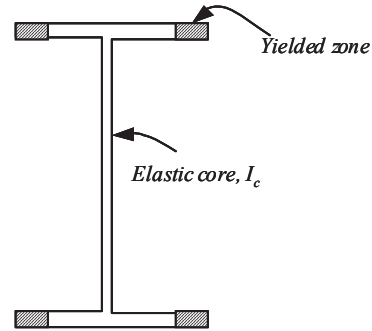
3.8.1 Inelastic Stiffness Reduction Factor – Modification

- This concept for calculating the effective length of columns in frames was widely accepted for many years.
- Over the past few years, a lot of modifications have been proposed to this method due to its several assumptions and limitation. Most of these modifications have not yet been accepted in to the AISC provisions.
- One of the accepted modifications is the inelastic stiffness reduction factor. As presented earlier, G is a measure of the *relative flexural rigidity* of the columns (EI_c/L_c) with respect to the beams (EI_b/L_b)

- However, if column buckling were to occur in the inelastic range ($\lambda_c < 1.5$), then the flexural rigidity of the column will be reduced because I_c will be the moment of inertia of only the elastic core of the entire cross-section. See figure below



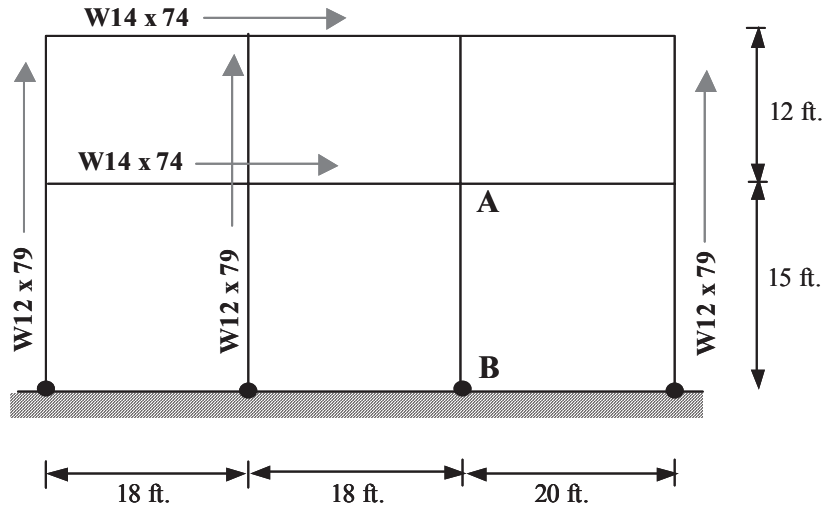
(a) Initial state – residual stress



(b) Partially yielded state at buckling

- The beams will have greater flexural rigidity when compared with the reduced rigidity (EI_c) of the inelastic columns. As a result, the beams will be able to restrain the columns better, which is good for column design.
- This effect is incorporated in to the AISC column design method through the use of Table 4-1 given on page 4-20 of the AISC manual.
- Table 4-1 gives the stiffness reduction factor (τ) as a function of the yield stress F_y and the stress P_u/A_g in the column, where P_u is factored design load (analysis)

EXAMPLE 3.7 Calculate the effective length factor for a W10 x 60 column AB made from 50 ksi steel in the unbraced frame shown below. Column AB has a design factor load $P_u = 450$ kips. The columns are oriented such that major axis bending occurs in the plane of the frame. The columns are braced *continuously along the length* for out-of-plane buckling. Assume that the same column section is used for the story above



Solution

Step I. Identify the frame type and calculate L_x , L_y , K_x , and K_y if possible.

- It is an unbraced (*sidesway uninhibited*) frame.
- $L_y = 0$ ft.
- K_y has no meaning because out-of-plane buckling is not possible.
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.

Step II (a) - Calculate K_x

- I_{xx} of W 14 x 74 = 796 in⁴ I_{xx} of W 10 x 60 = 341 in⁴

- $G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{341}{12 \times 12} + \frac{341}{15 \times 12}}{\frac{796}{18 \times 12} + \frac{796}{20 \times 12}} = \frac{4.2625}{7.002} = 0.609$
- $G_B = 10$ - for pin support, see note on Page 16.1-191
- Using G_A and G_B : $K_x = 1.8$ - from Alignment Chart on Page 16.1-192
- Note, K_x is greater than 1.0 because it is an unbraced frame.

Step II (b) - Calculate $K_{x-inelastic}$ using stiffness reduction factor method

- Reduction in the flexural rigidity of the column due to residual stress effects
 - First calculate, $P_u / A_g = 450 / 17.6 = 25.57$ ksi
 - Then go to Table 4-1 on page 4-20 of the manual, and read the value of stiffness reduction factor for $F_y = 50$ ksi and $P_u/A_g = 25.57$ ksi.
 - Stiffness reduction factor = $\tau = 0.833$
- $G_{A-inelastic} = \tau \times G_A = 0.833 \times 0.609 = 0.507$
- $G_B = 10$ - for pin support, see note on Page 16.1-191
- Using $G_{A-inelastic}$ and G_B , $K_{x-inelastic} = 1.75$ - alignment chart on Page 16.1-192
- Note: *You can combine Steps II (a) and (b) to calculate the $K_{x-inelastic}$ directly.* You don't need to calculate elastic K_x first. It was done here for demonstration purposes.
- Note that $K_{x-inelastic} < K_x$. This is in agreement with the fact that the beams offer better resistance to the *inelastic* column AB because it has reduced flexural rigidity.

Step III – Design strength of the column

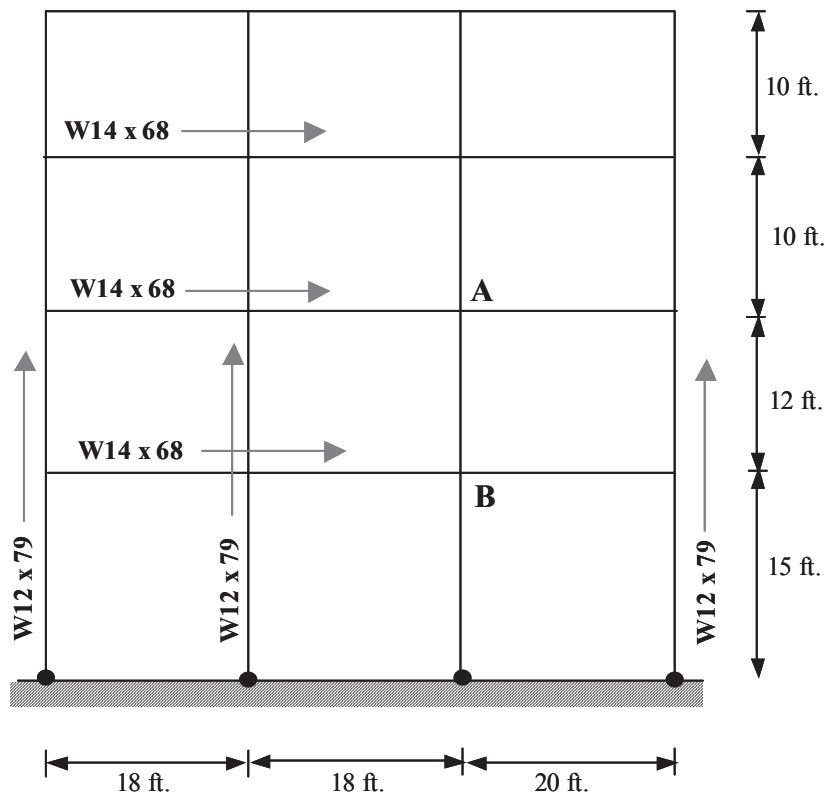
- $K_x L_x = 1.75 \times 15 = 26.25$ ft.
 - r_x / r_y for W10x60 = 1.71 - from Table 4-2, see page 4-26
 - $(KL)_{eq} = 26.25/1.71 = 15.35$ ft.

- $\phi_c P_n$ for X-axis buckling = 513.9 kips - from Table 4-2, see page 4-26
- Section slightly over-designed for $P_u = 450$ kips.

Column design strength = $\phi_c P_n = 513.9$ kips

EXAMPLE 3.8:

- Design Column AB of the frame shown below for a design load of 500 kips.
- Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame.
- Assume that the columns are braced at each story level for out-of-plane buckling.
- Assume that the same column section is used for the stories above and below.



Step I - Determine the design load and assume the steel material.

- Design Load = $P_u = 500$ kips
- Steel yield stress = 50 ksi (A992 material)

Step II. Identify the frame type and calculate L_x , L_y , K_x , and K_y if possible.

- It is an unbraced (sidesway uninhibited) frame.

- $L_x = L_y = 12$ ft.
- $K_y = 1.0$
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.
- Need to select a section to calculate K_x

Step III - Select a column section

- Assume minor axis buckling governs.
- $K_y L_y = 12$ ft.
- See Column Tables in AISC-LRFD manual
Select section W12x53
- $\phi_c P_n$ for y-axis buckling = 518 kips

Step IV - Calculate $K_{x-inelastic}$

- I_{xx} of W 12 x 53 = 425 in⁴ I_{xx} of W14x68 = 753 in⁴
- Account for the reduced flexural rigidity of the column due to residual stress effects
 - $P_u/A_g = 500 / 15.6 = 32.05$ ksi
 - Stiffness reduction factor = $\tau = 0.58$

$$G_A = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.58 \times \left(\frac{425}{10 \times 12} + \frac{425}{12 \times 12} \right)}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{3.766}{6.360} = 0.592$$

$$G_B = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.58 \times \left(\frac{425}{12 \times 12} + \frac{425}{15 \times 12} \right)}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{3.0812}{6.360} = 0.484$$

- Using G_A and G_B : $K_{x-inelastic} = 1.2$ - from Alignment Chart

Step V - Check the selected section for X-axis buckling

- $K_x L_x = 1.2 \times 12 = 14.4$ ft.
- r_x / r_y for W12x53 = 2.11

- Calculate $(KL)_{eq}$ to determine strength $(\phi_c P_n)$ for X-axis buckling
 $(KL)_{eq} = 14.4 / 2.11 = 6.825$ ft.
- From the column design tables, $\phi_c P_n$ for X-axis buckling = 612.3 kips

Step VI. Check the local buckling limits

For the flanges, $b_f/2t_f = 8.69 < \lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

For the web, $h/t_w = 28.1 < \lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$

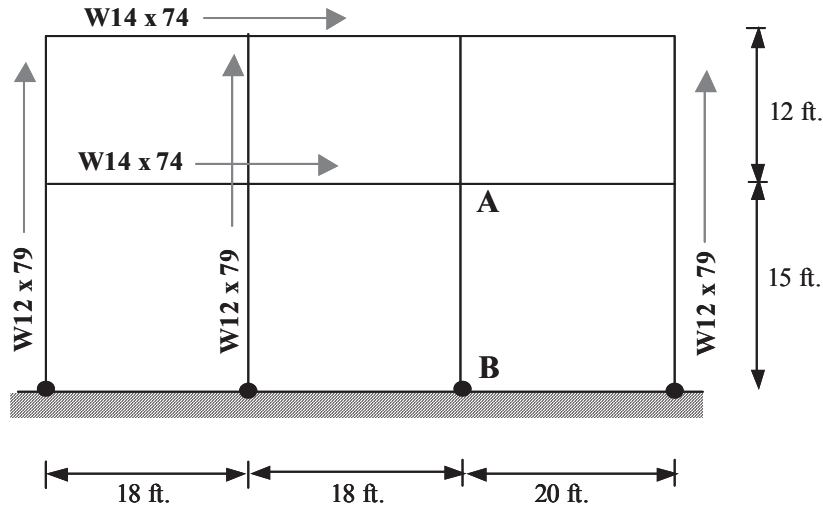
Therefore, the section is non-compact. OK, local buckling is not a problem

Step VII - Summarize the solution

$L_x = L_y = 12$ ft.	$K_y = 1.0$
$K_x = 1.2$ (inelastic buckling - sway frame-alignment chart method)	
$\phi_c P_n$ for Y-axis buckling = 518 kips	
$\phi_c P_n$ for X-axis buckling = 612.3 kips	
Y-axis buckling governs the design.	
Selected Section is W12 x 53 made from 50 ksi steel.	

EXAMPLE 3.9

- Design Column AB of the frame shown below for a design load of 450 kips.
- Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame.
- Assume that the columns are braced continuously along the length for out-of-plane buckling.
- Assume that the same column section is used for the story above.



Step I - Determine the design load and assume the steel material.

- Design Load = $P_u = 450$ kips
- Steel yield stress = 50 ksi

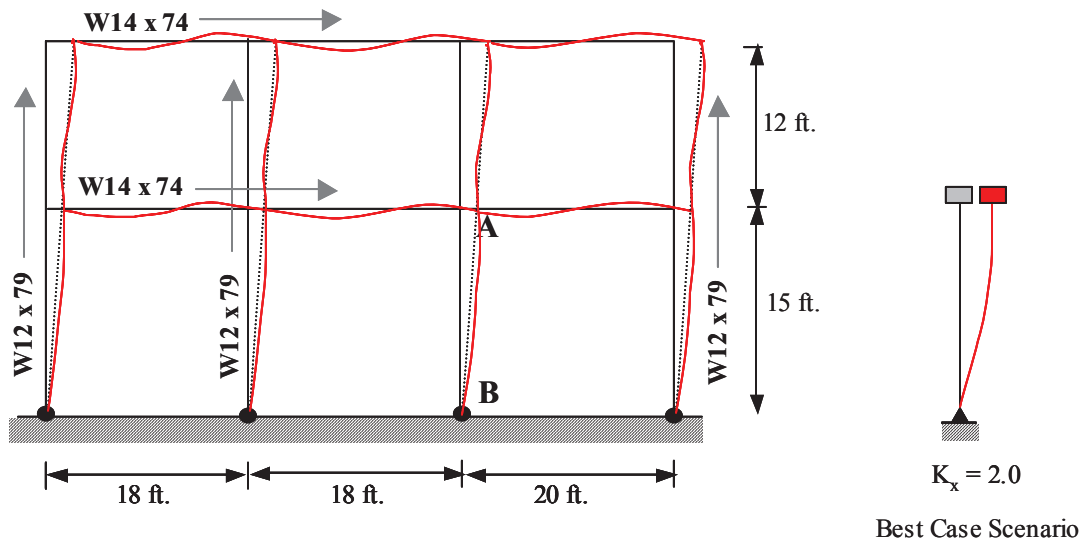
Step II. Identify the frame type and calculate L_x , L_y , K_x , and K_y if possible.

- It is an unbraced (*sidesway uninhibited*) frame.
- $L_y = 0$ ft.
- K_y has no meaning because out-of-plane buckling is not possible.
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.
- Need to select a section to calculate K_x

Step III. Select a section

- There is no help from the minor axis to select a section
- Need to assume K_x to select a section.

See Figure below:



- The best case scenario for K_x is when the beams connected at joint A have infinite flexural stiffness (rigid). In that case $K_x = 2.0$ from Table C-C2.1
- Actually, the beams don't have infinite flexural stiffness. Therefore, calculated K_x should be greater than 2.0.
- To select a section, assume $K_x = 2.0$
 - $K_x L_x = 2.0 \times 15.0 \text{ ft.} = 30.0 \text{ ft.}$
- Need to be able to calculate $(KL)_{eq}$ to be able to use the column design tables to select a section. Therefore, need to assume a value of r_x/r_y to select a section.
 - See the W10 column tables on page 4-26.
 - Assume $r_x/r_y = 1.71$, which is valid for W10 x 49 to W10 x 68.
- $(KL)_{eq} = 30.0/1.71 = 17.54 \text{ ft.}$
 - Obviously from the Tables, for $(KL)_{eq} = 17.5 \text{ ft.}$, W10 x 60 is the first section that will have $\phi_c P_n > 450 \text{ kips}$
- Select W10x60 with $\phi_c P_n = 457.7 \text{ kips}$ for $(KL)_{eq} = 17.5 \text{ ft.}$

Step IV - Calculate $K_{x\text{-inelastic}}$ using selected section

- I_{xx} of W 14 x 74 = 796 in⁴ I_{xx} of W 10 x 60 = 341 in⁴
- Account for the reduced flexural rigidity of the column due to residual stress effects
 - $P_u/A_g = 450 / 17.6 = 25.57$ ksi
 - Stiffness reduction factor = $\tau = 0.833$
- $$G_A = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.833 \times \left(\frac{341}{12 \times 12} + \frac{341}{15 \times 12} \right)}{\frac{796}{18 \times 12} + \frac{796}{20 \times 12}} = \frac{3.550}{7.002} = 0.507$$
- $G_B = 10$ - for pin support
- Using G_A and G_B : $K_{x\text{-inelastic}} = 1.75$ - from Alignment Chart on Page 3-6
- Calculate value of $K_{x\text{-inelastic}}$ is less than 2.0 (the assumed value) because G_B was assumed to be equal to 10 instead of ∞

Step V - Check the selected section for X-axis buckling

- $K_x L_x = 1.75 \times 15 = 26.25$ ft.
 - r_x / r_y for W10x60 = 1.71
 - $(KL)_{eq} = 26.25/1.71 = 15.35$ ft.
 - $(\phi_c P_n)$ for X-axis buckling = 513.9 kips
- Section slightly over-designed for $P_u = 450$ kips.
- *W10 x 54 will probably be adequate, Student should check by calculating K_x inelastic and $\phi_c P_n$ for that section.*

Step VI. Check the local buckling limits

For the flanges, $b_f/2t_f = 7.41$ $<$ $\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

For the web, $h/t_w = 18.7$ $<$ $\lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$

Therefore, the section is non-compact. OK, local buckling is not a problem

- **Step VII - Summarize the solution**

$L_y = 0 \text{ ft.}$ $K_y = \text{no buckling}$
$K_x = 1.75$ (inelastic buckling - sway frame - alignment chart method)
$\phi_c P_n$ for X-axis buckling = 513.9 kips
X-axis buckling governs the design.
Selected section is W10 x 60 (W10 x 54 will probably be adequate).

3.9 DESIGN OF SINGLY SYMMETRIC CROSS-SECTIONS

- So far, we have been talking about doubly symmetric wide-flange (I-shaped) sections and channel sections. These rolled shapes always fail by *flexural* buckling.
- Singly symmetric (Tees and double angle) sections fail either by *flexural* buckling about the axis of non-symmetry or by *flexural-torsional* buckling about the axis of symmetry and the longitudinal axis.

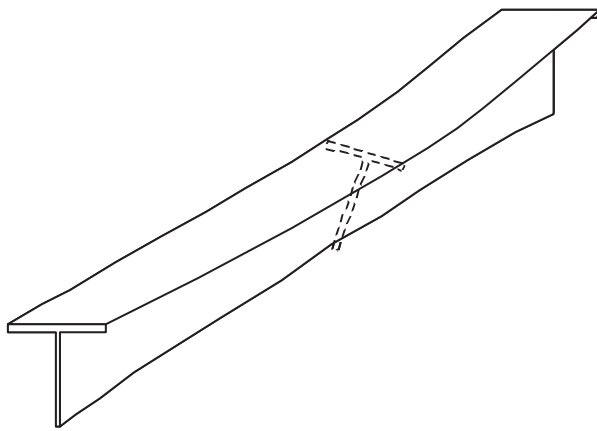


Figure 6(a). Flexural buckling

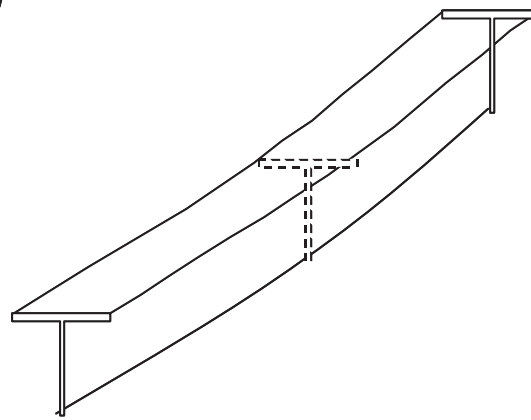


Figure 6(b). Flexural-torsional buckling

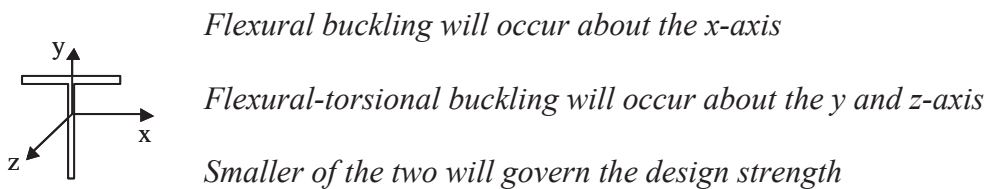


Figure 6(c). Singly symmetric cross-section

- The AISC specification for flexural-torsional buckling is given by Spec. E3.

$$\text{Design strength} = \phi_c P_n = 0.85 A_g F_{crft} \quad (1)$$

$$\text{Where, } F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2} \right) \left[1 - \sqrt{1 - \frac{4 F_{cry} F_{crz} H}{(F_{cry} + F_{crz})^2}} \right] \quad (2)$$

$$F_{cry} = \text{critical stress for buckling about the y-axis, see Spec. E2.} \quad (3)$$

$$F_{crz} = \frac{GJ}{A \bar{r}_o^2} \quad (4)$$

$$\bar{r}_o^2 = \text{polar radius of gyration about shear center (in.)} = y_o^2 + \frac{I_x + I_y}{A} \quad (5)$$

$$H = 1 - \frac{y_o^2}{\bar{r}_o^2} \quad (6)$$

$$y_o = \text{distance between shear center and centroid (in.)} \quad (7)$$

- The section properties for calculating the flexural-torsional buckling strength F_{crft} are given as follows:

- $G = \frac{E}{2(1+\nu)}$

- J, \bar{r}_o^2, H are given for WT shapes in Table 1-32 on page 1-101 to page 1-105

- \bar{r}_o^2, H are given for double-angle shapes in Table 1-35 on page 1-108 to 1-110

- J for single-angle shape in Table 1-31 on page 1-98 to 1-100. ($J_{2L} = 2 \times J_L$)

- The design tables for WT shapes given in Table 4-5 on page 4-35 to 4-47. These design tables include the axial compressive strength for flexural buckling about the x axis and flexural-torsional buckling about the y and z axis.

EXAMPLE 3.10 Calculate the design compressive strength of a WT10.5 x 66. The effective length with respect to x-axis is 25ft. 6in. The effective length with respect to the y-axis is 20 ft. and the effective length with respect to z-axis is 20ft. A992 steel is used.

Solution

- **Step I.** Buckling strength about x-axis

$$\lambda_{c-x} = \frac{K_x L_x}{r_x \pi} \sqrt{\frac{F_y}{E}} = \frac{306}{3.06 \times 3.1416} \sqrt{\frac{50}{29000}} = 1.321$$

$$\phi_c P_n = 0.85 \times (0.658)^{1.321^2} \times 50 \times 19.4 = 397.2 \text{ kips}$$

Values for A_g and r_x from page 4-41 of the manual. Compare with tabulated design strength for buckling about x-axis in Table 4-5

- **Step II.** Flexural-torsional buckling about the y and z axes

- Calculate F_{cry} and F_{crz} then calculate F_{crft} and $\phi_c P_n$

$$\lambda_{c-y} = \frac{K_y L_y}{r_y \pi} \sqrt{\frac{F_y}{E}} = \frac{240}{2.93 \times 3.1416} \sqrt{\frac{50}{29000}} = 1.083$$

$$F_{cry} = (0.658)^{1.083^2} \times 50 = 30.6 \text{ ksi}$$

$$F_{crz} = GJ/A \bar{I}_0^2 = 11,153 \times 5.62 / (4.60^2 \times 19.4) = 152.69$$

$$F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2H} \right) \left[1 - \sqrt{1 - \frac{4 F_{cry} F_{crz} H}{(F_{cry} + F_{crz})^2}} \right] = \left(\frac{30.6 + 152.7}{2 \times 0.844} \right) \left[1 - \sqrt{1 - \frac{4 \times 30.6 \times 152.7 \times 0.844}{(30.6 + 152.7)^2}} \right]$$

$$F_{crft} = 108.58 \times 0.272 = 29.534 \text{ ksi}$$

$$\phi_c P_n = 0.85 \times F_{crft} \times A_g = 0.85 \times 29.534 \times 19.4 = 487 \text{ kips}$$

Values for J , \bar{r}_o^2 , and H were obtained from flexural-torsional properties given in Table 1-32 on page 1-102. Compare the $\phi_c P_n$ value with the value reported in Table 4-5 (page 4-41) of the AISC manual.

- **Step III.** Design strength and check local buckling

Flanges: $b_f/2t_f = 12.4/(2 \times 1.03) = 6.02$, which is $< \lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

Stem of Tee: $d/t_w = 10.9/0.65 = 16.77$, which is $< \lambda_r = 0.75 \times \sqrt{\frac{E}{F_y}} = 18.08$

Local buckling is not a problem. Design strength = 397.2 kips. X-axis flexural buckling governs.

3.10 DESIGN OF DOUBLE ANGLE SECTIONS

- Double-angle sections are very popular as compression members in trusses and bracing members in frames.
 - These sections consist of two angles placed back-to-back and connected together using bolts or welds.
 - You have to make sure that the two single angle sections are connected such that they do not buckle (individually) between the connections along the length.
 - The AISC specification E4.2 requires that $\mathbf{Ka/r_z}$ of the individual single angles $< \frac{3}{4}$ of the governing $\mathbf{KL/r}$ of the double angle.
 - where, a is the distance between connections and r_z is the smallest radius of gyration of the single angle (see dimensions in Table 1-7)
- Double-angle sections can fail by flexural buckling about the x-axis or flexural torsional buckling about the y and z axes.

- For flexural buckling about the x-axis, the moment of inertia I_{x-2L} of the double angle will be equal to two times the moment of inertia I_{x-L} of each single angle.
- For flexural torsional buckling, there is a slight problem. The double angle section will have some *additional flexibility* due to the intermittent connectors. This added flexibility will depend on the connection parameters.
- According to AISC Specification E4.1, a modified $(KL/r)_m$ must be calculated for the double angle section for buckling about the y-axis to account for this added flexibility

- Intermediate connectors that are snug-tight bolted $\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_z}\right)^2}$

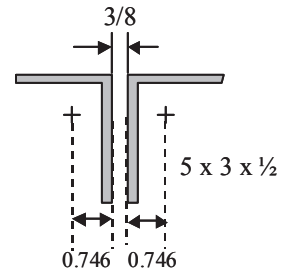
- Intermediate connectors that are welded or fully tensioned bolted:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82 \frac{\alpha^2}{1 + \alpha^2} \left(\frac{a}{r_y}\right)^2}$$

where, $\alpha = \text{separation ratio} = h/2r_y$

$h = \text{distance between component centroids in the y direction}$

EXAMPLE 3.11 Calculate the design strength of the compression member shown in the figure. Two angles, 5 x 3 x 1/2 are oriented with the long legs back-to-back and separated by 3/8 in. The effective length KL is 16 ft. A36 steel is used. Assume three welded intermediate connectors



Solution

Step I. Determine the relevant properties from the AISC manual

Property	Single angle	Double angle
A_g	3.75 in ²	7.5 in ²
r_x	1.58 in.	1.58 in.
r_y	0.824 in.	1.24 in.
r_z	0.642 in.	-----
J	0.322 in ⁴	0.644 in ⁴
\bar{r}_o^2		2.51 in.
H		0.646
AISC Page no.	1-36, 1-37, 1-99	1-75, 1-109

Step II. Calculate the x-axis buckling strength

- $KL/r_x = 16 \times 12 / 1.58 = 120.8$
- $\lambda_{c-x} = \frac{K_x L_x}{r_x \pi} \sqrt{\frac{F_y}{E}} = \frac{120.8}{3.1416} \sqrt{\frac{36}{29000}} = 1.355$
- $\phi_c P_n = 0.85 \times (0.658)^{1.355^2} \times 36 \times (2 \times 3.75) = 106 \text{ kips}$

Step III. Calculate $(KL/r)_m$ for y-axis buckling

- $(KL/r) = 16 \times 12 / 1.24 = 154.8$

- $a/r_z = 48/0.648 = 74.07$
 $a/r_z = 74.07 < 0.75 \times KL/r = 0.75 \times 154.8 = 115.2$ (OK!)
- $\alpha = h/2r_y = (2 \times 0.75 + 0.375)/(2 \times 0.829) = 1.131$
- $\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82 \frac{\alpha^2}{1 + \alpha^2} \left(\frac{a}{r_y}\right)^2}$
 $= \sqrt{(154.8)_o^2 + 0.82 \frac{1.131^2}{1 + 1.131^2} \left(\frac{48}{0.829}\right)^2} = 158.5$

Step IV. Calculate flexural torsional buckling strength.

- $\lambda_{c-y} = \left(\frac{KL}{r}\right)_m \times \frac{1}{\pi} \times \sqrt{\frac{F_y}{E}} = 1.778$
- $F_{cry} = \frac{0.877}{\lambda_{c-y}^2} \times F_y = \frac{0.877}{1.778^2} \times 36 = 9.987$ ksi
- $F_{crz} = \frac{GJ}{A\bar{r}_o^2} = \frac{11,200 \times 0.644}{7.5 \times 2.51^2} = 151.4$ ksi
- $F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2H}\right) \left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right] = \left(\frac{9.987 + 151.4}{2 \times 0.646}\right) \left[1 - \sqrt{1 - \frac{4 \times 9.987 \times 151.4 \times 0.646}{(9.987 + 151.4)^2}}\right]$
 $F_{crft} = 9.748$ ksi
- $\phi_c P_n = 0.85 \times F_{crft} \times A_g = 0.85 \times 9.748 \times 7.50 = 62.1$ kips

Flexural torsional buckling strength controls. The design strength of the double angle member is 62.1 kips.

Step V. Compare with design strengths in Table 4-10 (page 4-84) of the AISC manual

- $\phi_c P_n$ for x-axis buckling with unsupported length = 16 ft. = 106 kips
- $\phi_c P_n$ for y-z axis buckling with unsupported length = 16 ft. = 61.3 kips

These results make indicate excellent correlation between the calculations in steps II to IV and the tabulated values.

Design tables for double angle compression members are given in the AISC manual. See Tables 4-9, 4-10, and 4-11 on pages 4-78 to 4-93

- In these Tables $F_y = 36$ ksi
- Back to back distance = $3/8$ in.
- Design strength for buckling about x axis
- Design strength for flexural torsional buckling accounting for the *modified* slenderness ratio depending on the number of intermediate connectors.
- These design Tables can be used to design compression members as double angle sections.