7.1.3 Geometry of Horizontal Curves

The horizontal curves are, by definition, circular curves of radius R. The elements of a horizontal curve are shown in Figure 7.9 and summarized (with units) in Table 7.2.

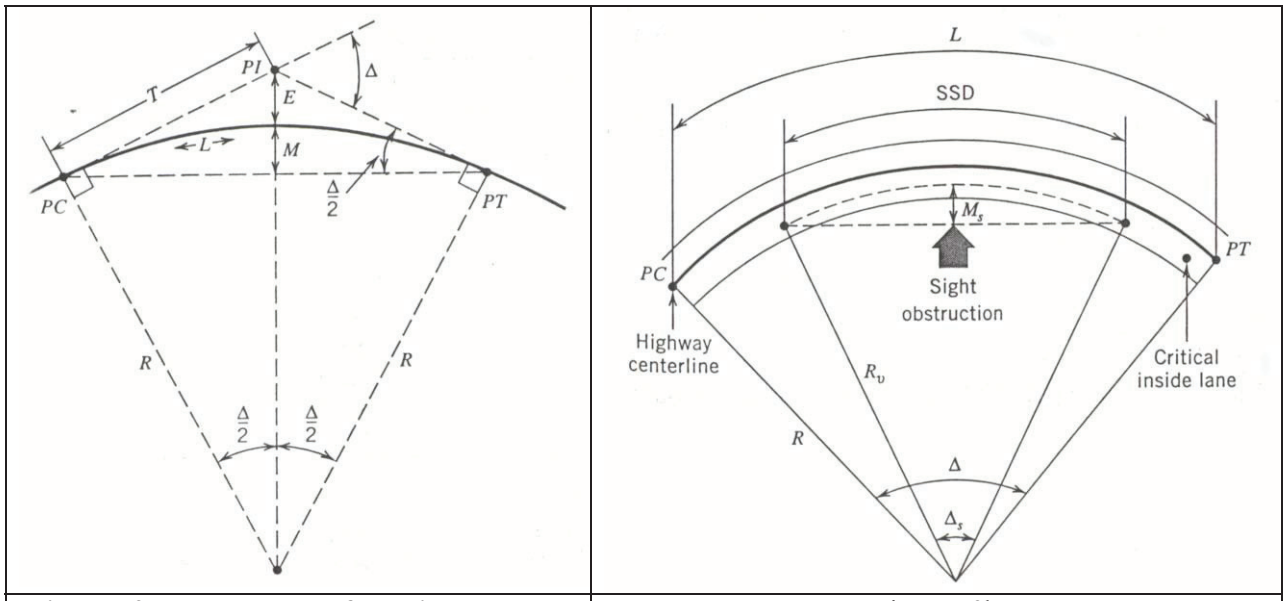

Figure 7.9a The elements of a horizontal curve	Figure 7.9b

Table 7.2 A summary of horizontal curve elements

Symbol	Name	Units
PC	Point of curvature, start of horizontal curve	
PT	Point of tangency, end of horizontal curve	
PI	Point of tangent intersection	
D	Degree of curvature	degrees per 100 feet of centerline
R	Radius of curve (measured to centerline)	feet
L	Length of curve (measured along centerline)	feet
Δ	Central (subtended) angle of curve, PC to PT	degrees
T	Tangent length	feet
M	Middle ordinate	feet
LC	Length of long chord, from PC to PT	feet
E	External distance	feet

The equations 7.8 through 7.13 that apply to the analysis of the curve are given below.

$$D = \frac{36{,}000}{2\pi R} = \frac{5729.6}{R} \tag{7.8}$$

$$L = \frac{100\,\Delta}{D} \tag{7.9}$$

$$T = R \tan \frac{1}{2}\Delta \tag{7.10}$$

$$M = R\left(1 - \cos \frac{1}{2}\Delta\right) \tag{7.11}$$

$$LC = 2R \sin \frac{1}{2}\Delta \tag{7.12}$$

$$E = R\left(\frac{1}{\cos \frac{1}{2}\Delta} - 1\right) \tag{7.13}$$

Example 7.5

A 7-degree horizontal curve covers an angle of 63°15'34". Determine the radius, the length of the curve, and the distance from the circle to the chord M.

Solution to Example 7.5

Rearranging Equation 7.8,with D = 7 degrees, the curve's radius R can be computed. Equation 7.9 allows calculation of the curve's length L, once the curve's central angle is converted from 63°15'34" to 63.2594 degrees. The middle ordinate calculation uses Equation 7.11. These computations are shown below.

$$R = \frac{5729.6}{7} = 818.5\text{feet}$$

$$L = \frac{100 \times 63.2594°}{7} = 903.7\text{feet}$$

$$M = 818.5 * (1 - \cos 31.6297°) = 121.6\text{feet}$$

If metric units are used, the definition of the degree of the curve must be carefully examined. Because the definition of the degree of curvature D is the central angle subtended by a 100-foot arc, then a "metric D" would be the angle subtended by a 30.5-meter arc. The subtended angle Δ does not change, but the metric values of R, L, and M become

$$R = \frac{5729.6}{7 * 3.28} = 249.55 \text{ meters}$$

$$\Delta = 63.2594^{o}; \quad \frac{1}{2}\Delta = 31.6297$$

$$L = \frac{100 * 63.2594°}{7 * 3.28} = 275.52 \text{ meters}$$

$$M = 249.55 * (1 - \cos 31.6297°) = 37.07 \text{ meters}$$