Fluid dynamics - Equation of continuity and Bernoulli's principle.

Fluid statics

•What is a fluid?
Density
Pressure
•Fluid pressure and depth
Pascal's principle
•Buoyancy
Archimedes' principle

Fluid dynamics
•Reynolds number
•Equation of continuity
•Bernoulli's principle
•Viscosity and turbulent flow
•Poiseuille's equation

Lecture 4
Dr Julia Bryant

web notes: Fluidslect4.pdf

flow1.pdf flow2.pdf

http://www.physics.usyd.edu.au/teach_res/jp/fluids/wfluids.htm

Fluid dynamics

REYNOLDS NUMBER

A British scientist Osborne Reynolds (1842 – 1912) established that the nature of the flow depends upon a *dimensionless* quantity, which is now called the Reynolds number R_e.

$$R_e = \rho v L / \eta$$

ρ density of fluid

v average flow velocity over the cross section of the pipe

L characteristic dimension

η viscosity

$$R_e = \rho\, v\, L / \eta$$

$$[R_e] \equiv \frac{[kg.m^{-3}]\,[m.s^{-1}][m]}{[Pa.s]}$$

$$\equiv \frac{kg}{m^3} \times \frac{m}{s} \times m \times \frac{s^2.m^2}{kg.m.s} = [1]$$

R_e is a dimensionless number

pascal	1 Pa = 1 $N.m^{-2}$
newton	1 N = 1 $kg.m.s^{-2}$

1 Pa.s = $kg.m.s^{-2}$. m^{-2} .s

As a *rule of thumb,* for a flowing fluid

R_e < ~ **2000** laminar flow

~ 2000 < R_e < ~ 3000 unstable laminar to turbulent flow

R_e > ~ **2000** turbulent flow

Consider an IDEAL FLUID

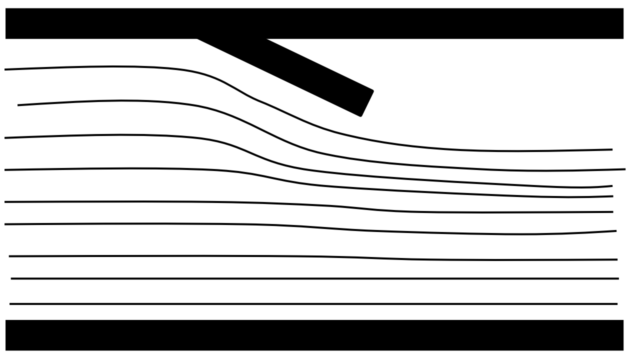
Fluid motion is very complicated. However, by making some assumptions, we can develop a useful model of fluid behaviour. An ideal fluid is

Incompressible – the density is constant
Irrotational – the flow is smooth, no turbulence
Nonviscous – fluid has no internal friction ($\eta=0$)
Steady flow – the velocity of the fluid at each point is constant in time.

Consider the average motion of the fluid at a particular point in space and time.

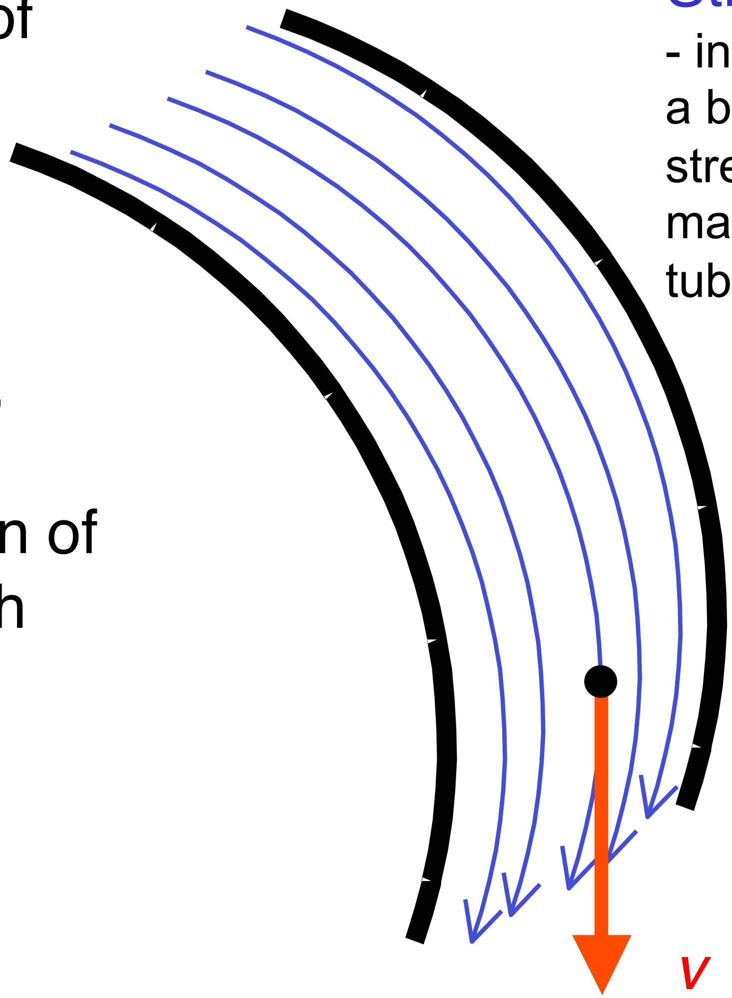
An individual fluid element will follow a path called a *flow line*.

Steady flow is when the pattern of flow lines does not change with time.



Streamlines

- in steady flow, a bundle of streamlines makes a flow tube.



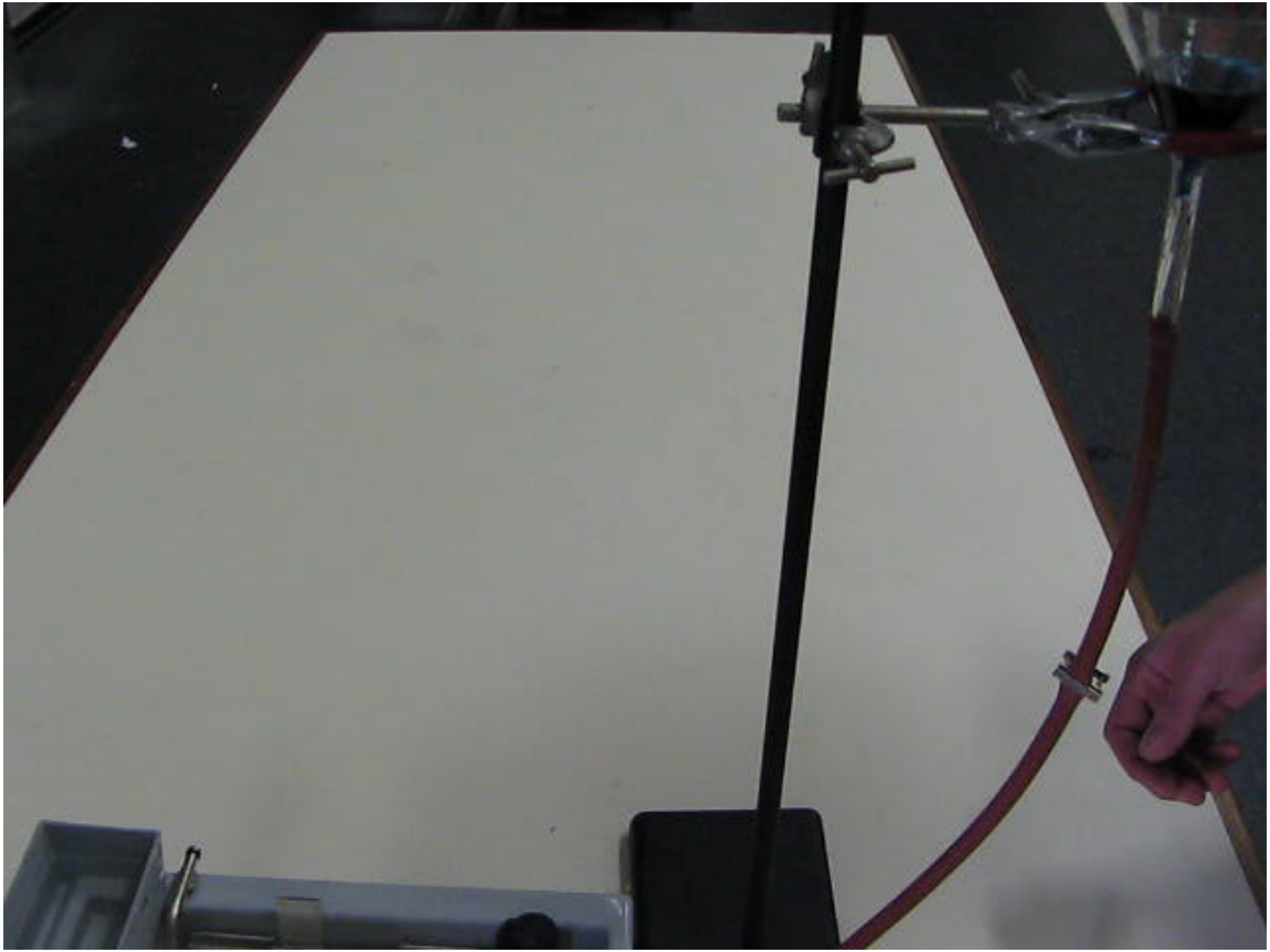
Velocity of particle is tangent to streamline

Streamlines cannot cross

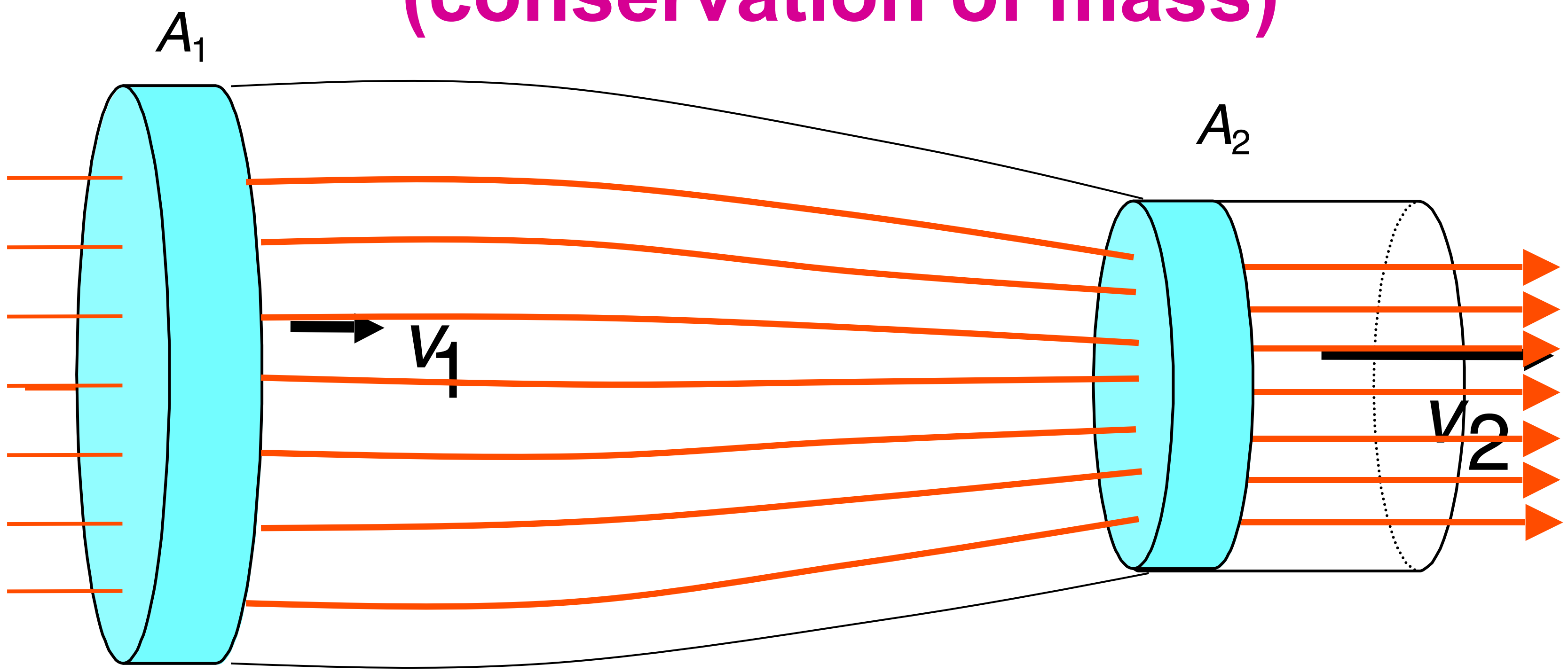
Steady flow is also called laminar flow.

If steady flow hits a boundary or if the rate of flow increases, steady flow can become chaotic and irregular. This is called turbulent flow.

For now, we will consider only steady flow situations.



A model **EQUATION OF CONTINUITY (conservation of mass)**



The mass of fluid in a flow tube is constant.

Where streamlines crowd together the flow speed increases.

EQUATION OF CONTINUITY (conservation of mass)

mass flowing in = mass flowing out

$$m_1 = m_2$$

$$\rho V_1 = \rho V_2$$

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 = Q = V/t = \text{constant}$$

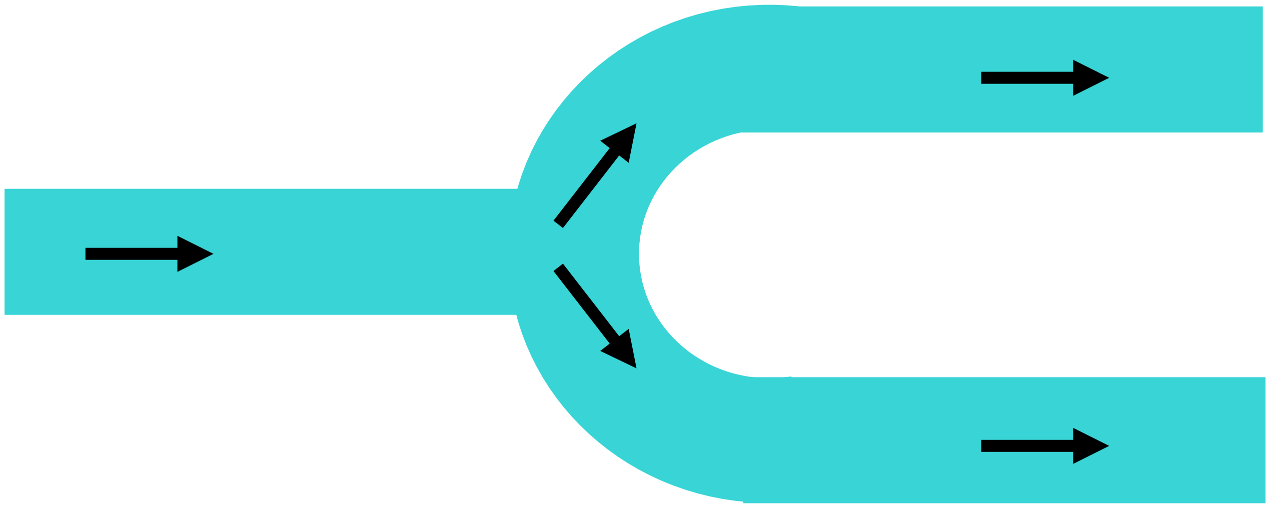
$V = A\,\Delta x$ $\quad v = \Delta x/\Delta t$
So $V = A\,v\Delta t$

Q=volume flow rate m^3s^{-1}

Applications

Rivers
Circulatory systems
Respiratory systems
Air conditioning systems

Y-junction with pipes of the same diameter



$$A_i\, v_i = A_f\, v_f$$

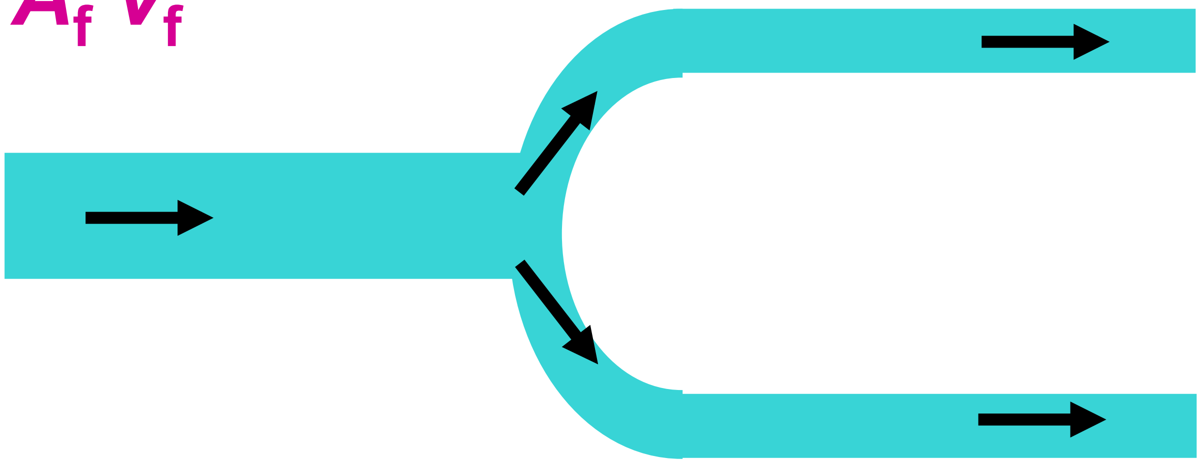
$$A_f = 2A_i$$

So flow speed must drop to half

$$v_f = 1/2\, v_i \text{ ms}^{-1}$$

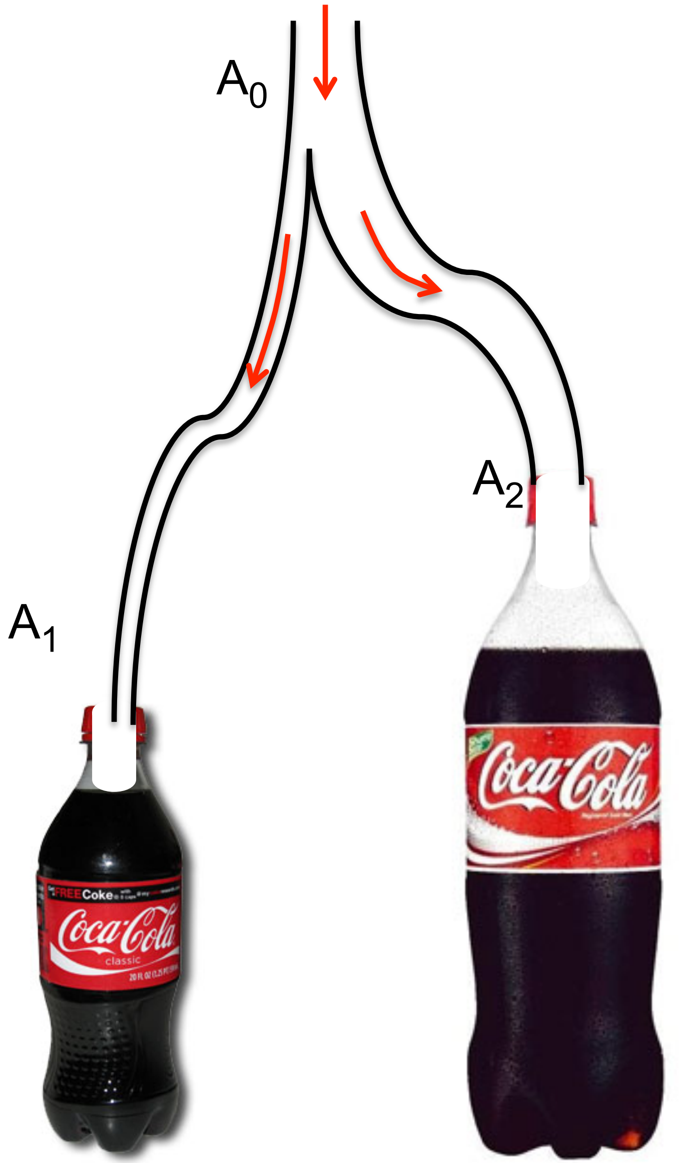
Y-junction with pipes of half the diameter

$$A_i\, v_i \;=\; A_f\, v_f$$



The pipes after the branch must have half the cross-sectional area of those before if the flow speed is to stay the same.

How big does each of the tubes need to be if the different sized coke bottles are to fill in the same time?



375 ml

1.25 L

If the velocity is constant,

$(A_1 + A_2)\,v = A_0\,v$

Volume flow rate is

$V/t = Av \quad m^3s^{-1}$

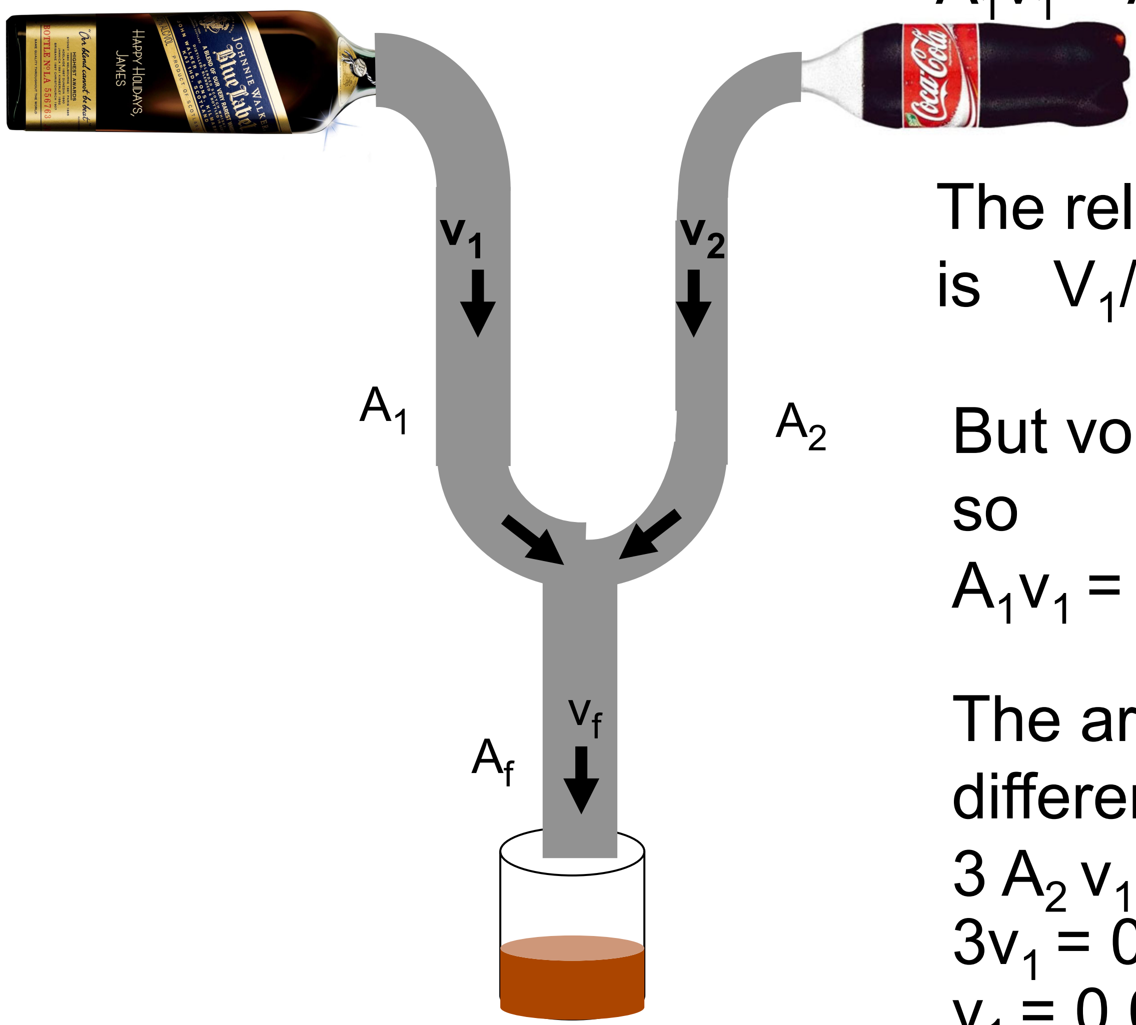
Bottles fill in time t

$$t = \frac{V}{Av} = \frac{0.375}{A_1\cancel{v}} = \frac{1.25}{A_2\cancel{v}} \ s \qquad \boxed{A_1 = A_0 - A_2}$$

$$0.375.\,A_2 = 1.25\,A_0 - 1.25\,A_2$$

$$A_2 = 1.25\,A_0 / 1.625\ m^2$$
$$= 0.77\,A_0\ m^2$$
$$A_1 = 0.23\,A_0\ m^2$$

What velocity must the flow of scotch and coke be to make a drink with the volume of scotch being 15% that of the coke, if the scotch tube is 3 times the area of the coke tube?



From the continuity equation

$A_1v_1 + A_2v_2 = A_fv_f$

The relative volume flow rate is $V_1/t_1 = 0.15\ V_2/t_2\ m^3s^{-1}$

But volume flow rate $V/t = Av$ so

$A_1v_1 = 0.15\ A_2v_2\ m^3s^{-1}$

The areas of the pipes are different $A_1 = 3\ A_2\ m^2$

$3\ A_2\ v_1 = 0.15\ A_2v_2$

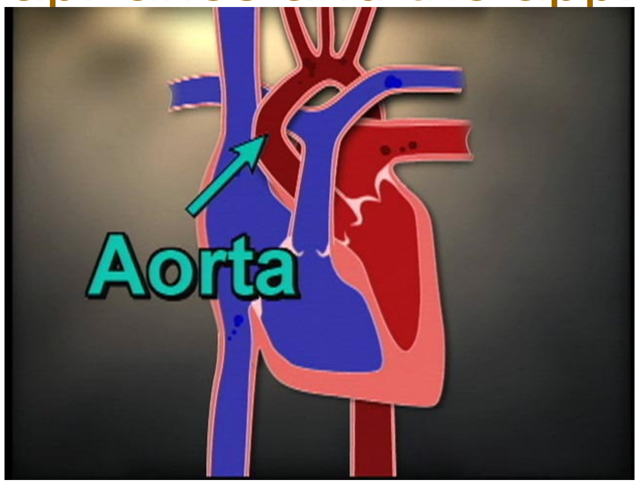
$3v_1 = 0.15\ v_2$

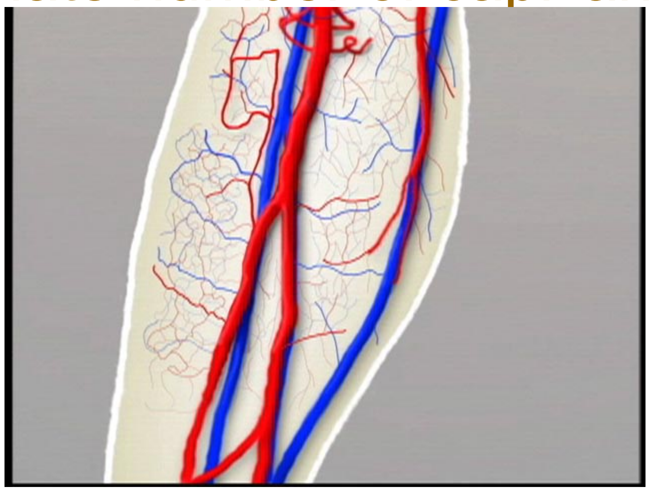
$v_1 = 0.05\ v_2\ m.s^{-1}$

Blood flowing through our body

The radius of the aorta is ~ 10 mm and the blood flowing through it has a speed ~ 300 mm.s^{-1}. A capillary has a radius ~ 4×10^{-3} mm but there are literally billions of them. The average speed of blood through the capillaries is ~ 5×10^{-4} m.s^{-1}.

Calculate the effective cross sectional area of the capillaries and the approximate number of capillaries.





Setup

aorta $R_A = 10\times10^{-3}$ m =10mm … and A_A=cross sectional area of aorta

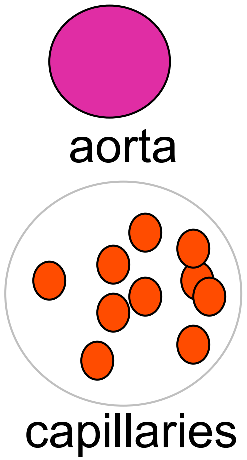
capillaries $R_C = 4\times10^{-6}$ m =0.004mm … and A_C =cross sectional area of capillaries

aorta $v_A = 0.300$ m.s^{-1}

capillaries $v_C = 5\times10^{-4}$ m.s^{-1}

Assume steady flow of an ideal fluid and apply the equation of continuity

$$Q = A\,v = \text{constant} \Rightarrow A_A\,v_A = A_C\,v_C$$



Area of capilliaries A_C

$$A_C = A_A\,(v_A / v_C) = \pi\,R_A^{\,2}\,(v_A / v_C)$$

$$A_C = 0.20 \text{ m}^2$$

Number of capillaries N

$$A_C = N\,\pi\,R_C^{\,2}$$

$$N = A_C / (\pi\,R_C^{\,2}) = 0.2 / \{\pi\,(4\times10^{-6})^2\}$$

$$N = 4\times10^9$$

BERNOULLI'S PRINCIPLE

An increase in the speed of fluid flow results in a decrease in the pressure.
(In an ideal fluid.)

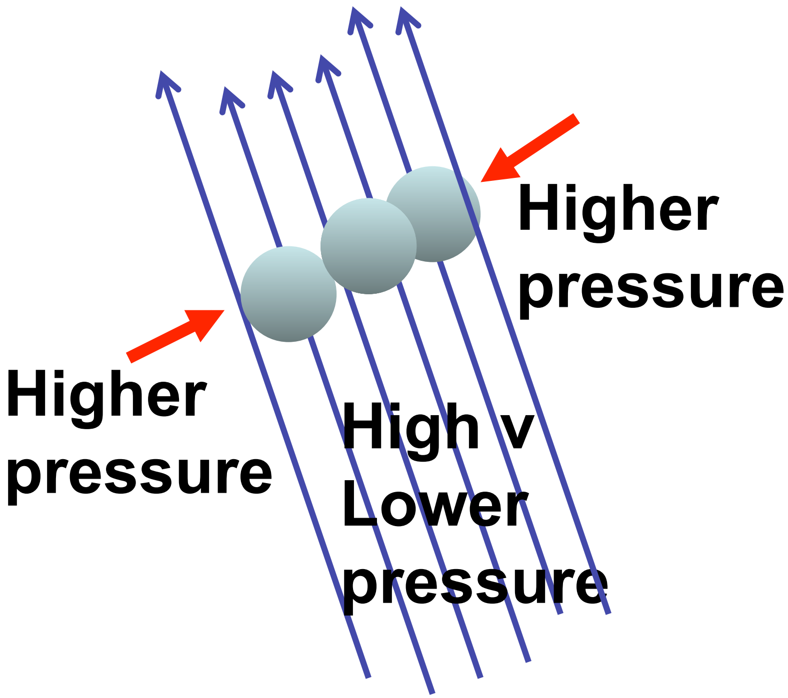
Daniel Bernoulli
(1700 – 1782)

How can a plane fly?
What is the venturi effect?
Why is Bernoulli's principle handy in a bar?
Why does a cricket ball swing or a baseball curve?

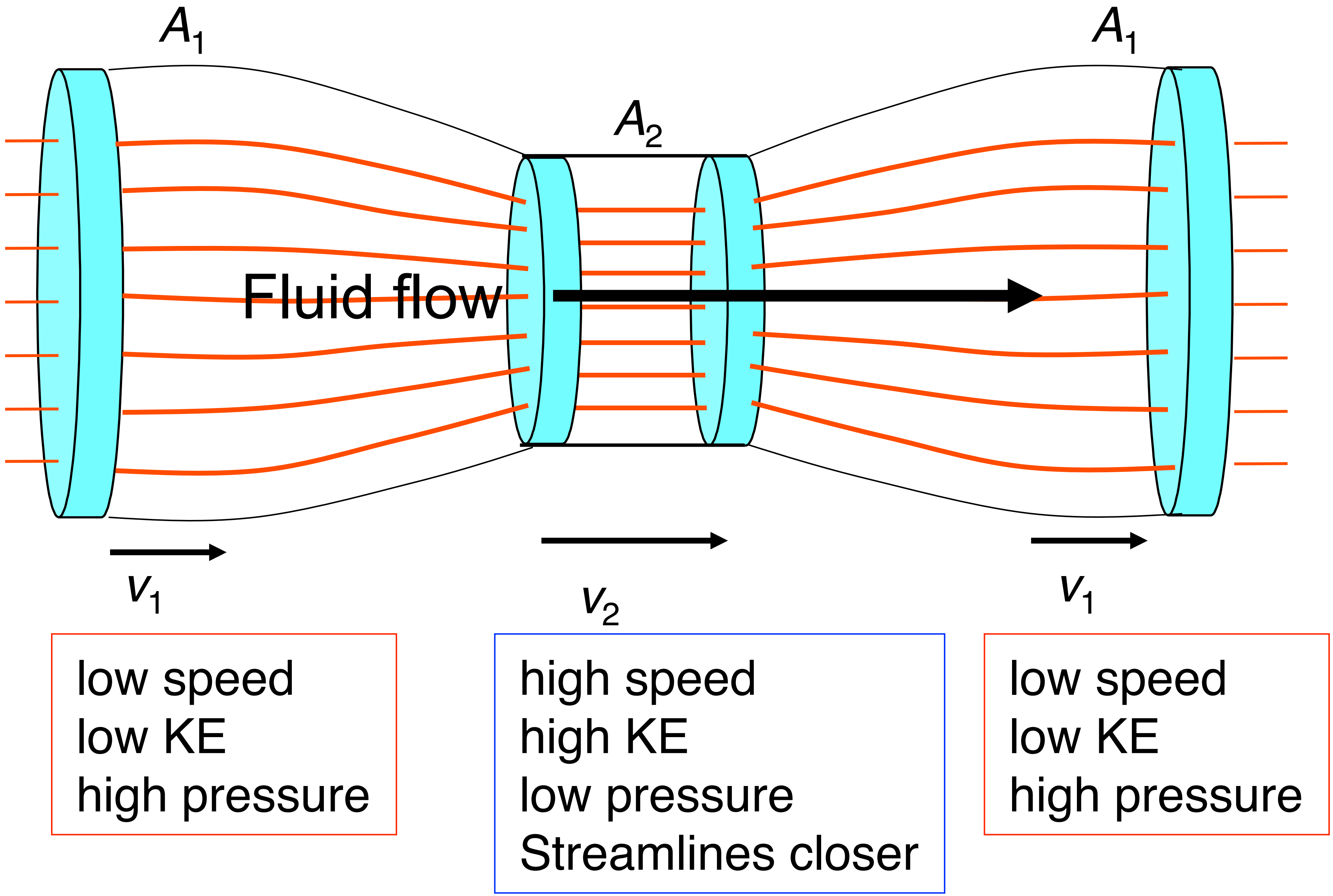
web notes:
flow3.pdf

Bernoulli's floating ball

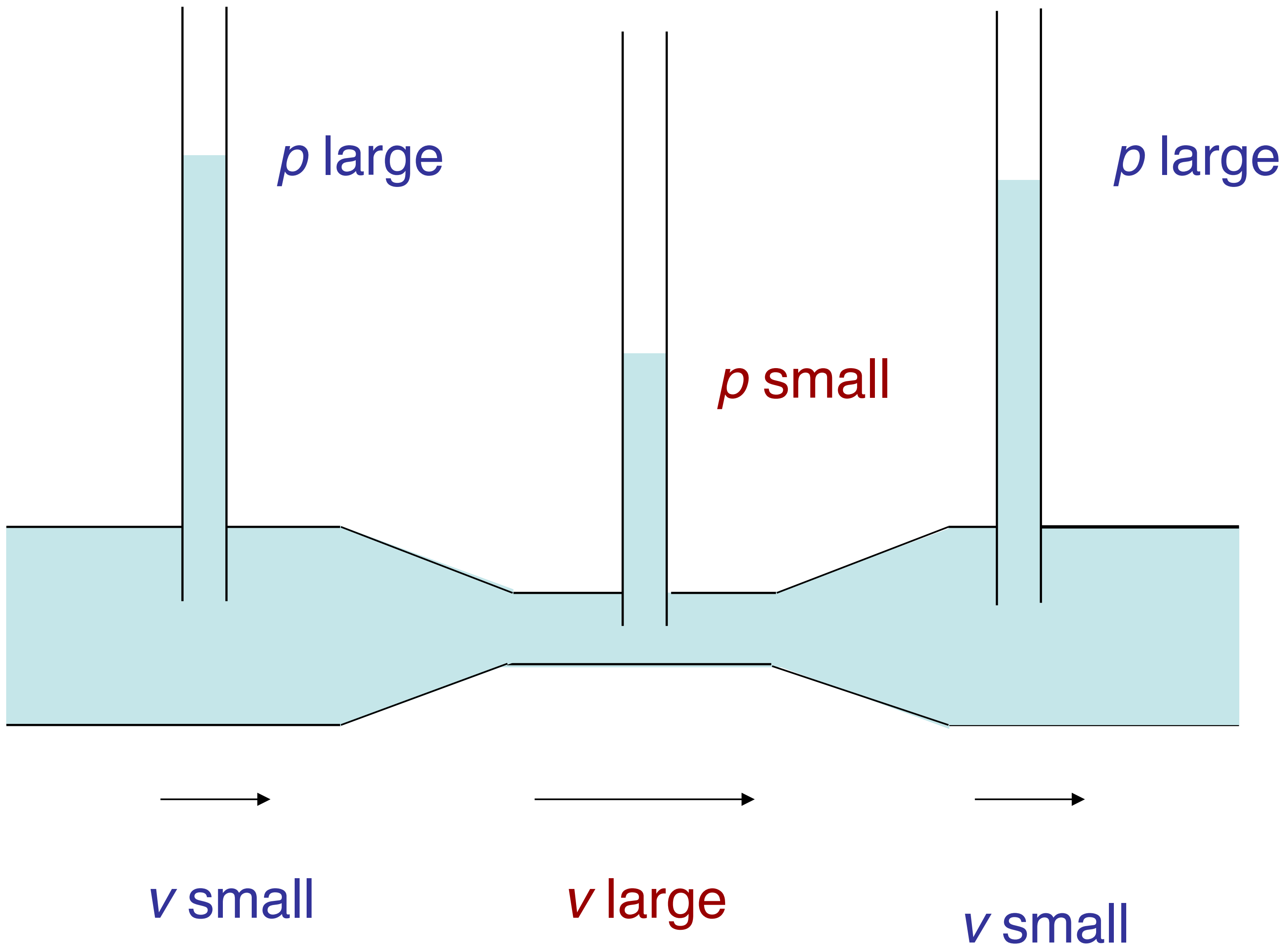
- the vacuum cleaner shop experiment!



DEMO



A_1
A_2
A_1
Fluid flow
v_1
v_2
v_1
low speed
low KE
high pressure
high speed
high KE
low pressure
Streamlines closer
low speed
low KE
high pressure



p large
p small
p large
v small
v large
v small

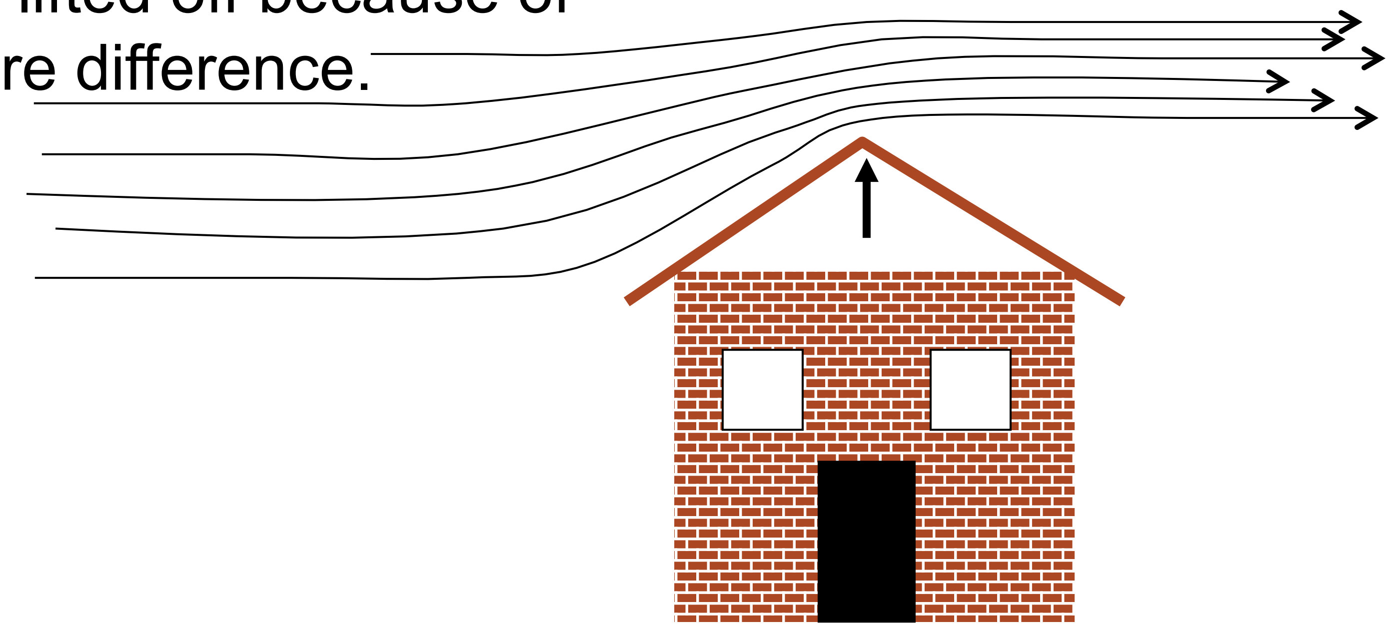
In a storm how does a house lose its roof?

Air flow is disturbed by the house. The "streamlines" crowd around the top of the roof.

⇒faster flow above house

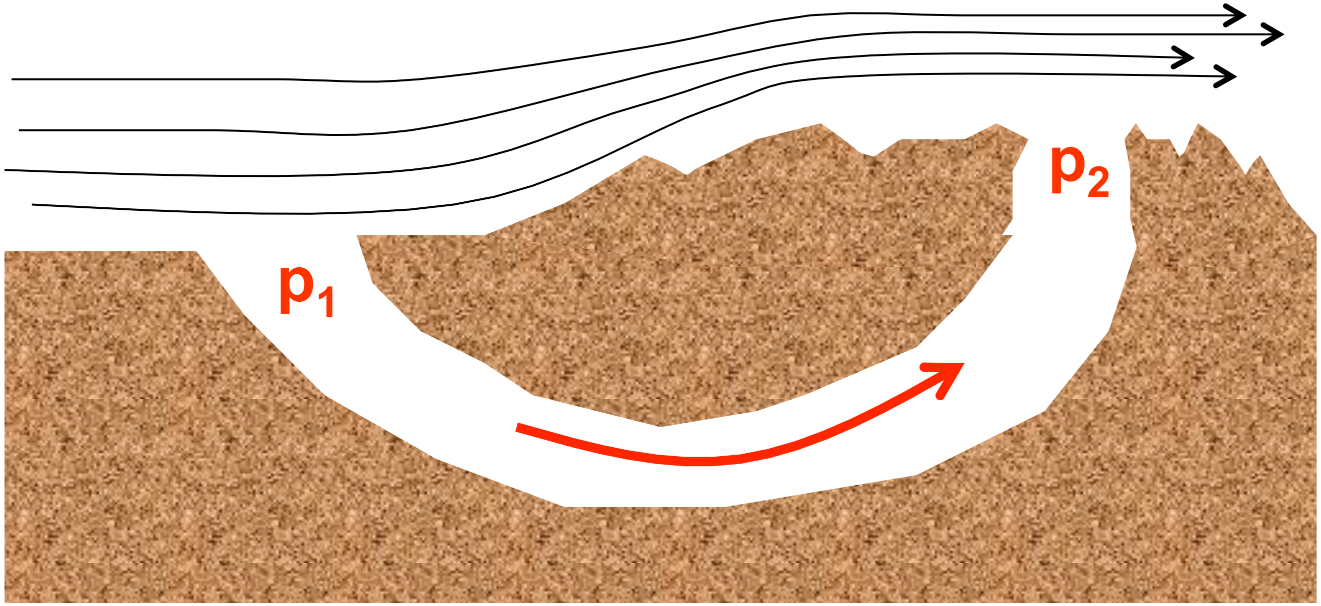
⇒reduced pressure above roof to that inside the house

⇒roof lifted off because of pressure difference.



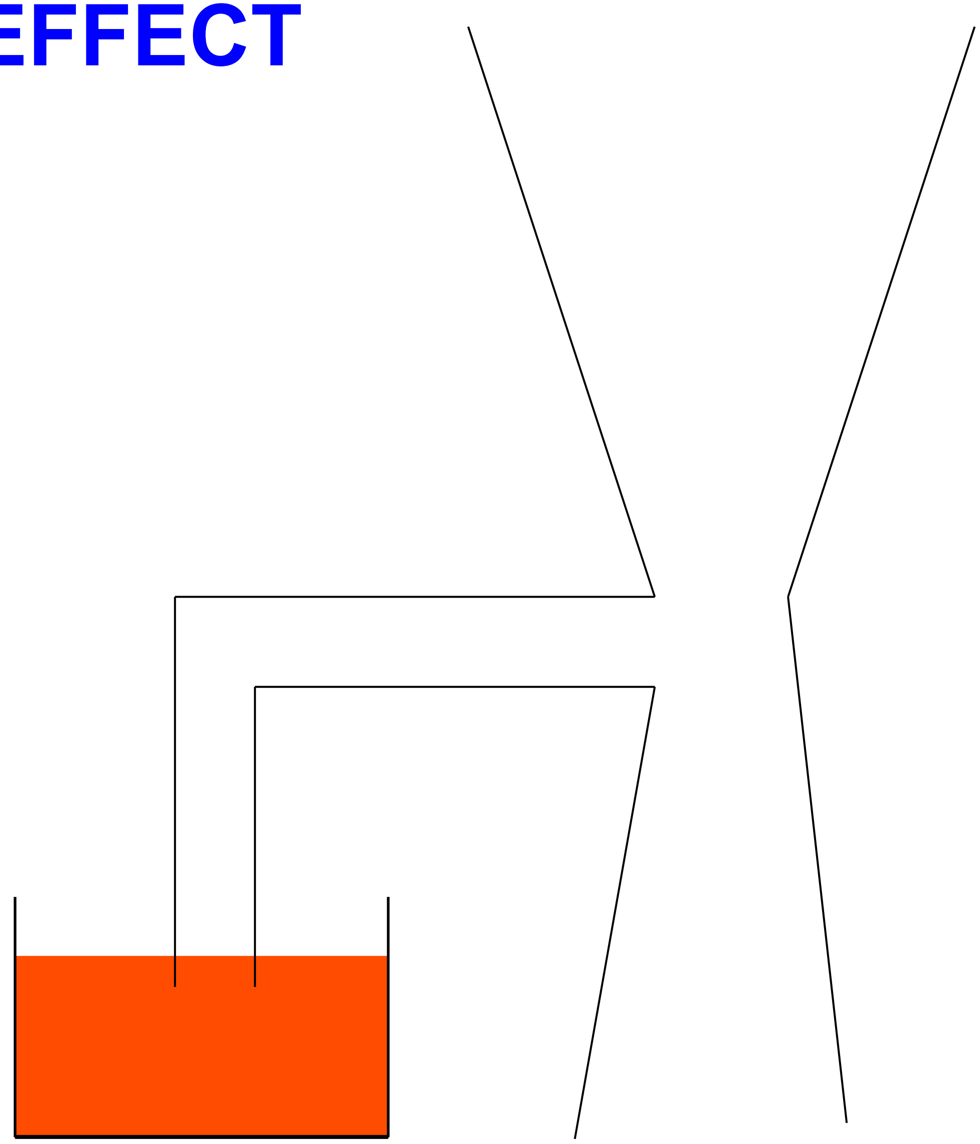
Why do rabbits not suffocate in the burrows?

Air must circulate. The burrows must have two entrances.
Air flows across the two holes is usually slightly different
$\Rightarrow$ slight pressure difference
$\Rightarrow$ forces flow of air through burrow.
One hole is usually higher than the other and the a small mound is built around the holes to increase the pressure difference.



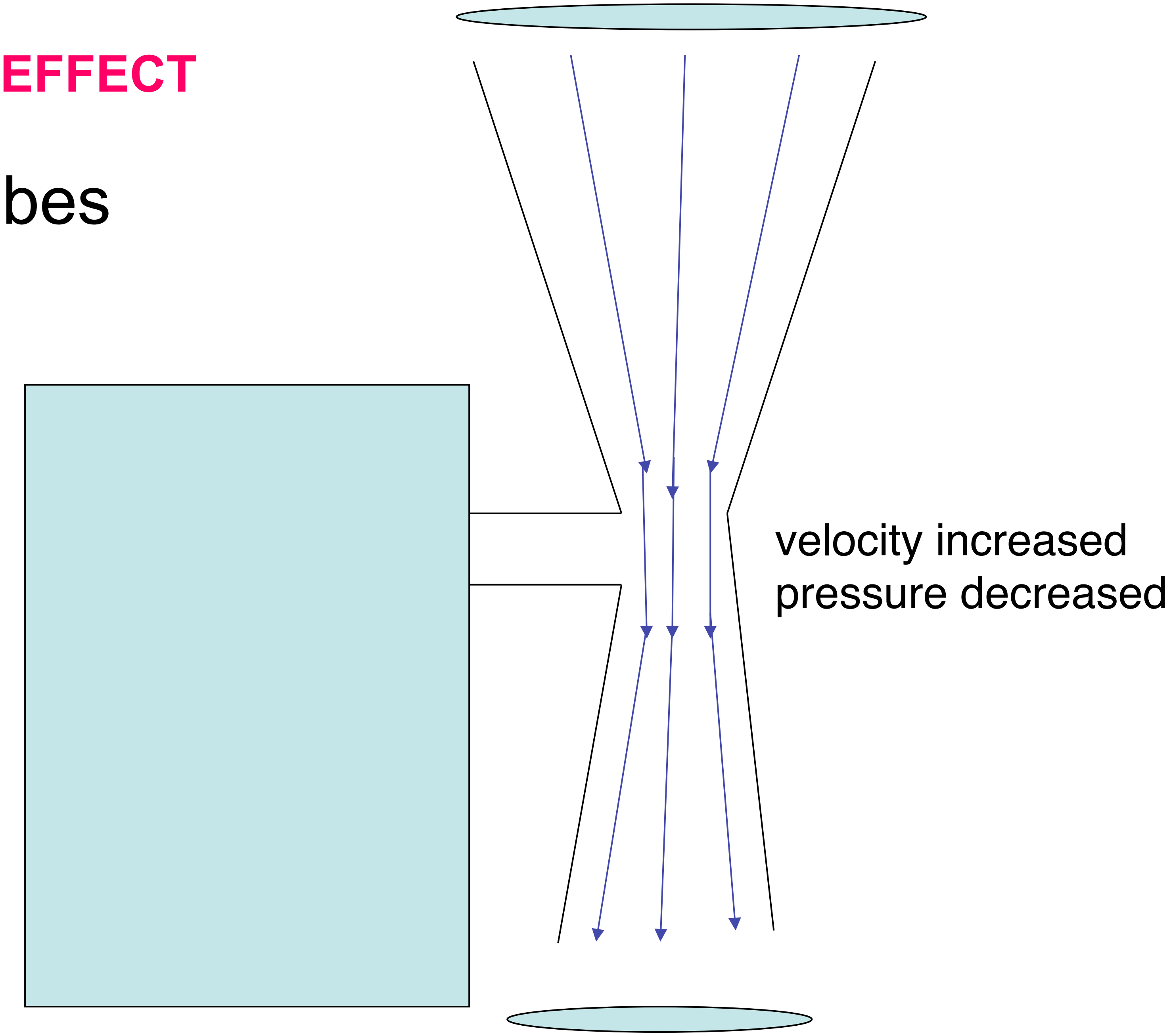
$p_2 < p_1$

VENTURI EFFECT



VENTURI EFFECT

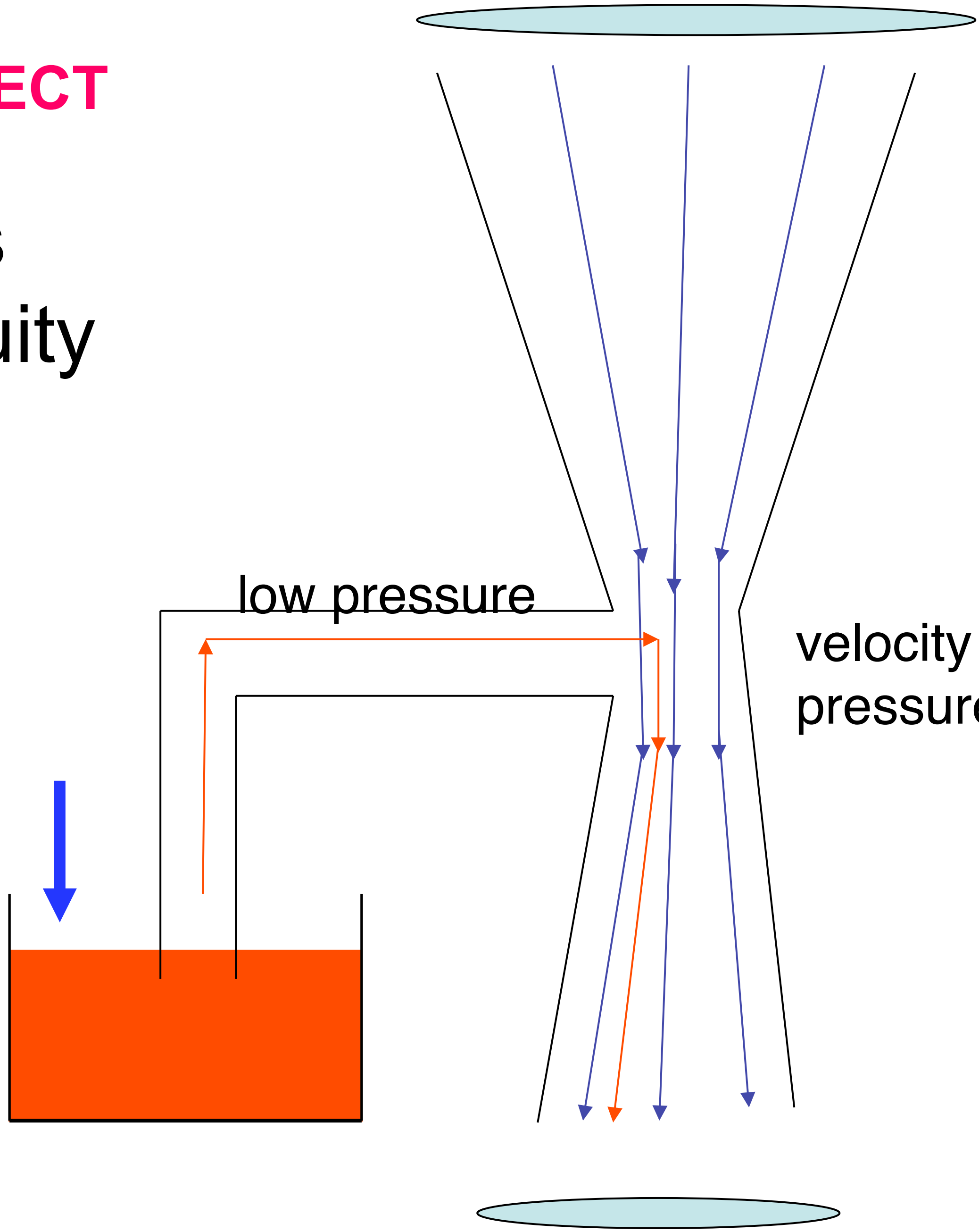
Flow tubes



VENTURI EFFECT

Flow tubes
Eq Continuity

high pressure (p_{atm})

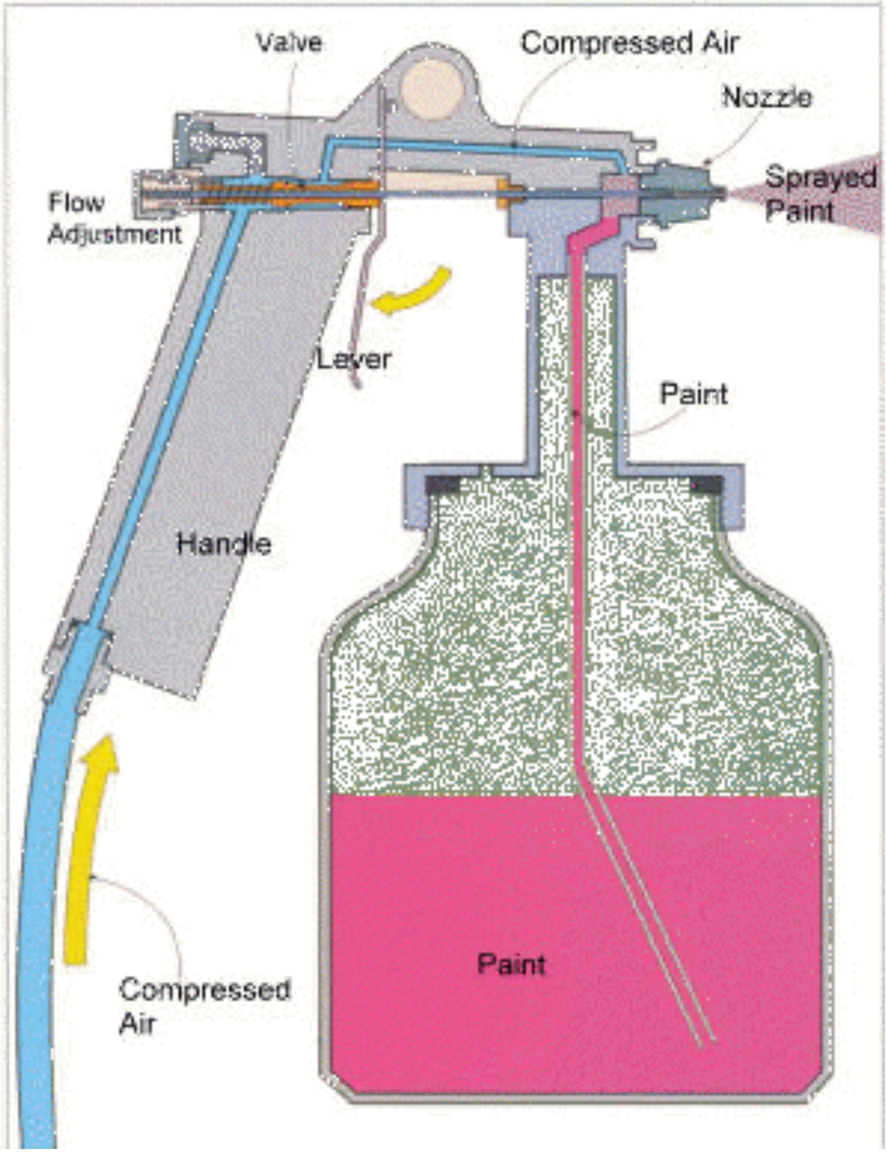


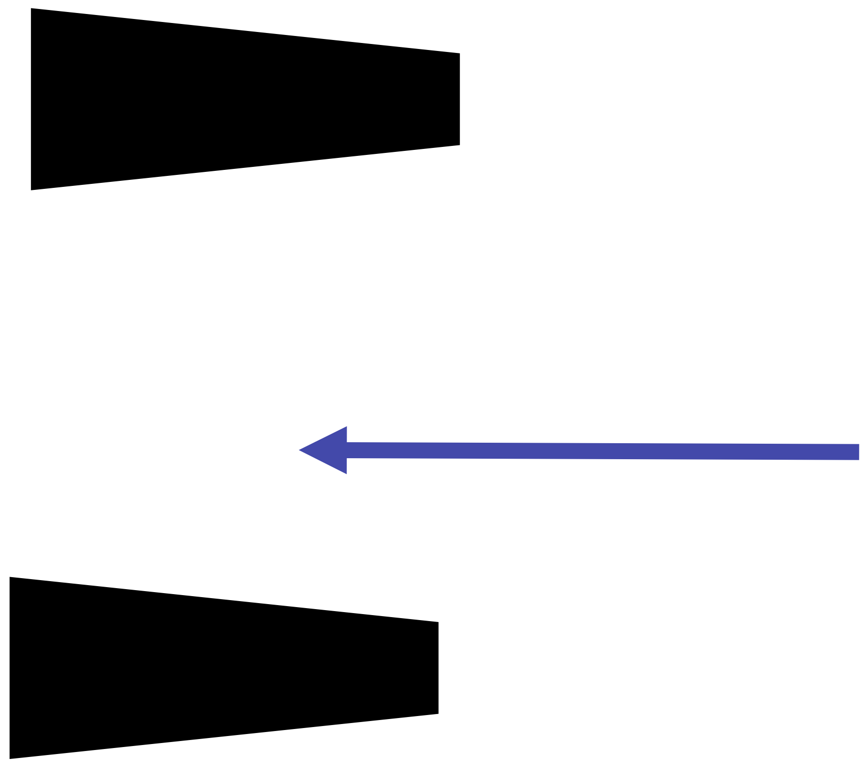
velocity increased
pressure decreased

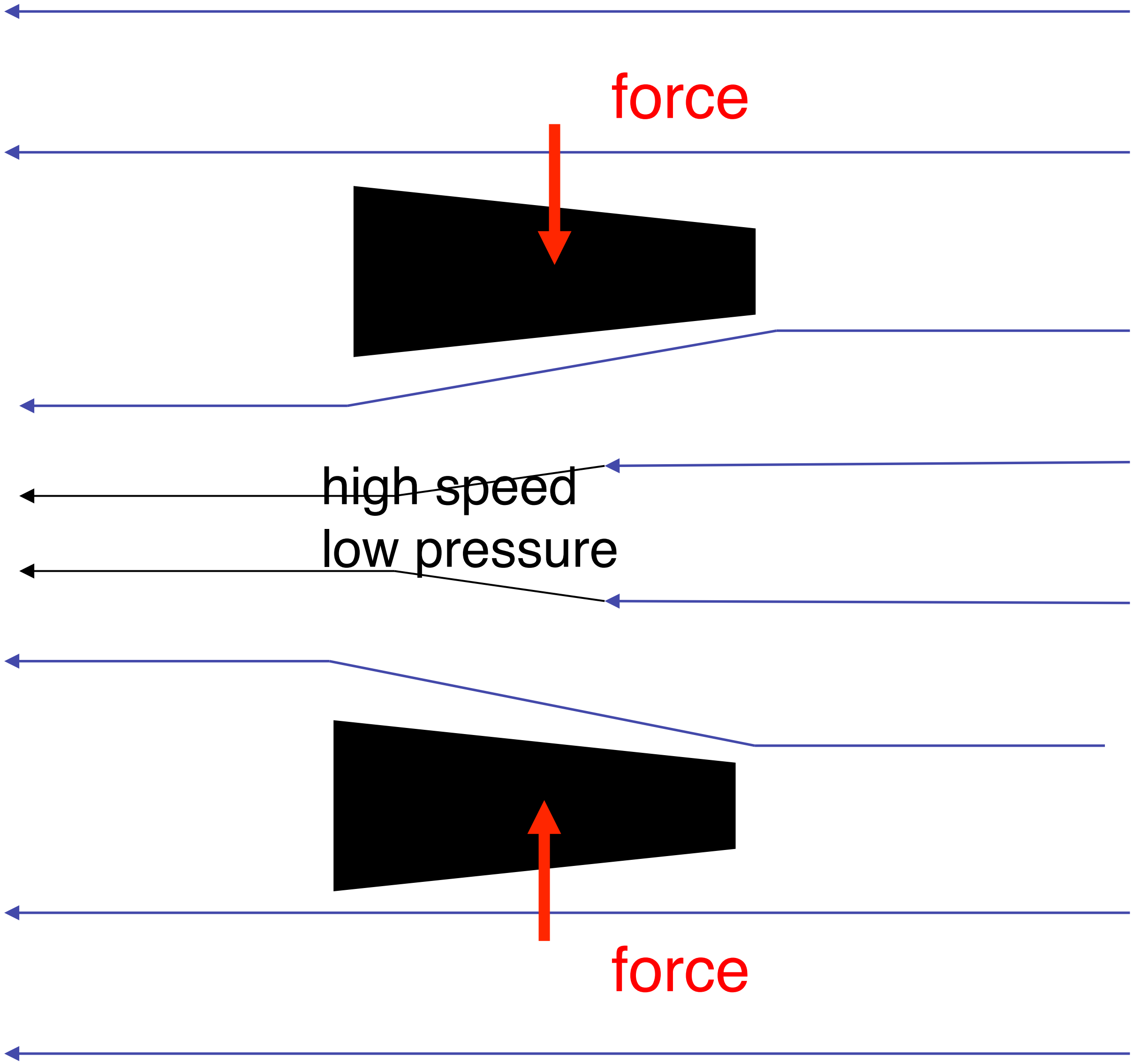
DEMO

VENTURI EFFECT

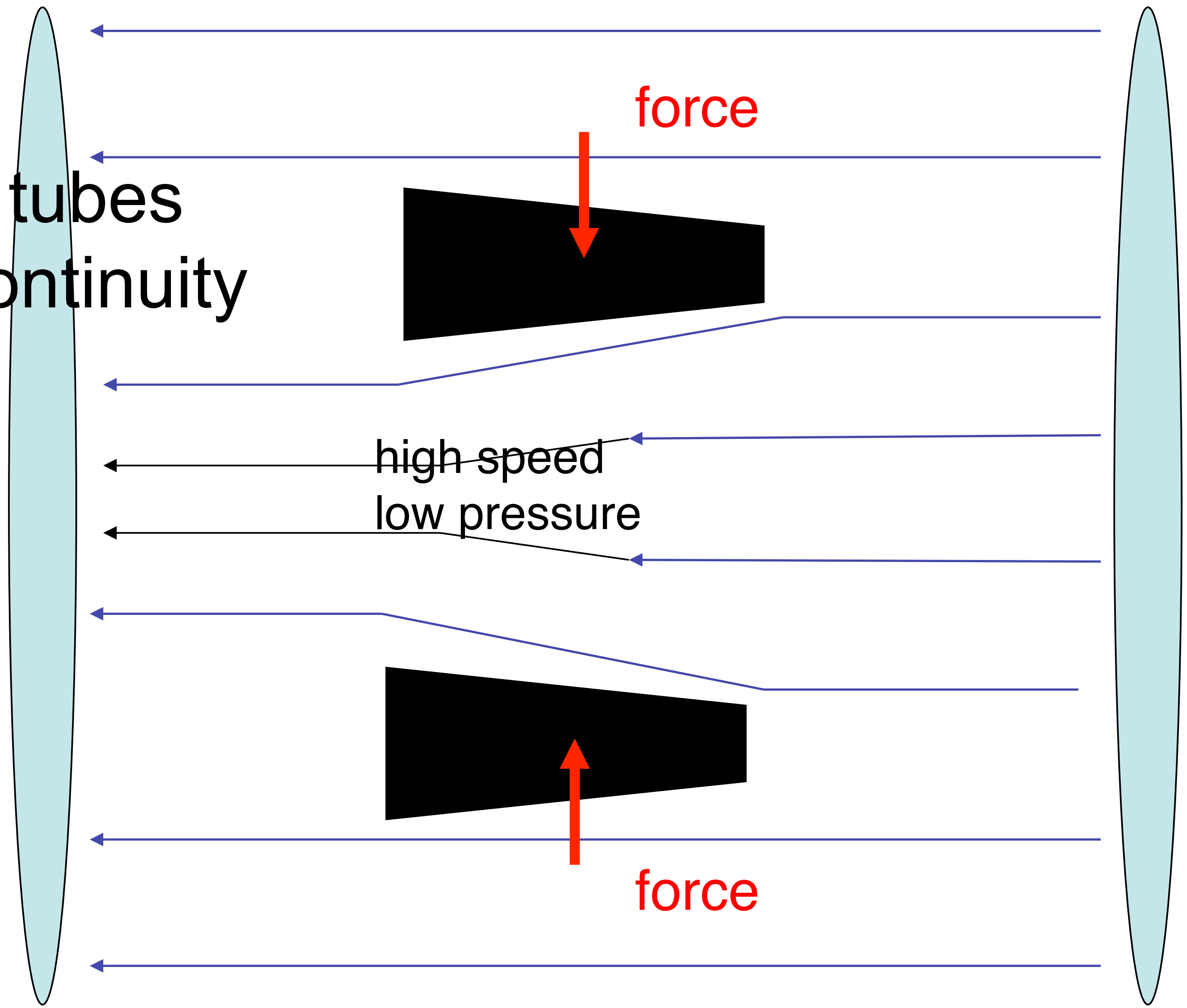
Spray gun



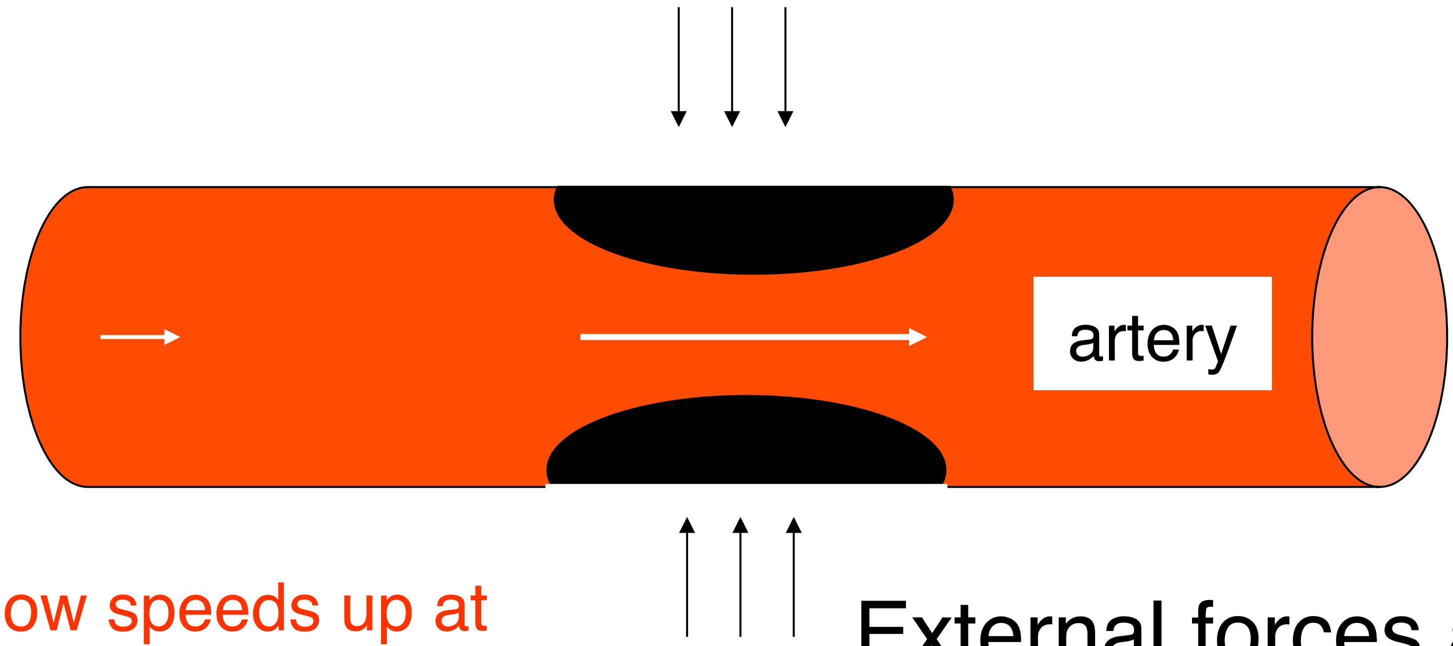



force
high speed
low pressure
force

Flow tubes
Eq continuity



What happens when two ships or trucks pass alongside each other?
Have you noticed this effect in driving across the Sydney Harbour Bridge?



Flow speeds up at constriction

Pressure is lower

Internal force acting on artery wall is reduced

External forces are unchanged

Artery can collapse

Arteriosclerosis and vascular flutter

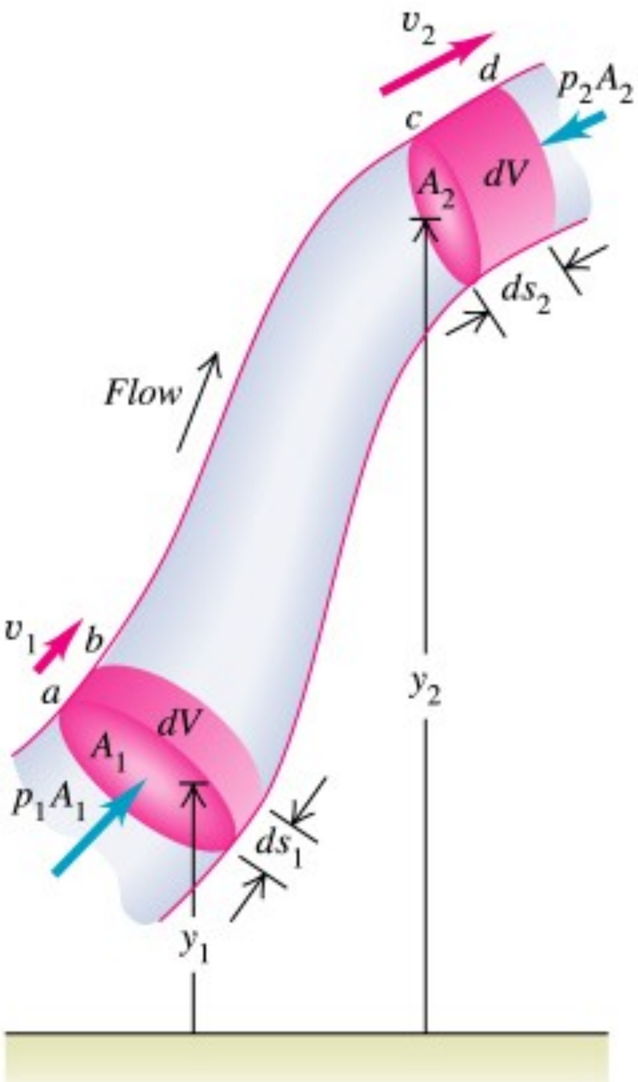
Bernoulli’s Equation

Consider an element of fluid with uniform density.

The change in energy of that element as it moves along a pipe must be zero - conservation of energy.

This is the basis for Bernoulli’s equation.

Continuity



Volume passing from point c to d in time *dt* is
$dV = A_2 ds_2$

Volume passing from point a to b in time *dt* is
$dV = A_1 ds_1$

$$dV = A_1 ds_1 = A_2 ds_2 \quad \text{(Continuity)}$$

Derivation of Bernoulli's equation

Consider conservation of energy

Work done = kinetic energy + potential energy

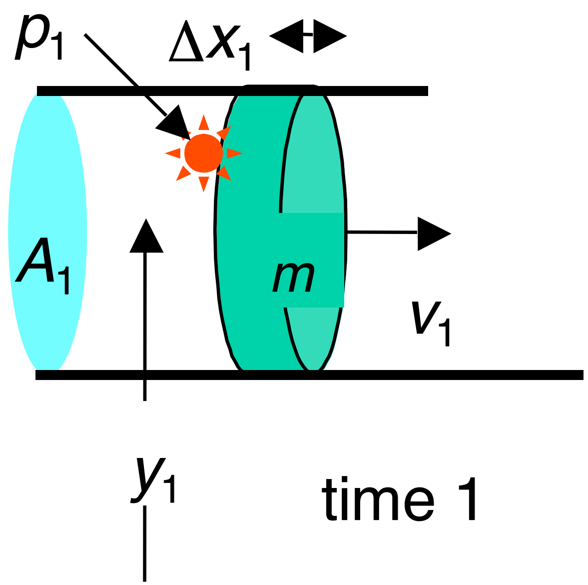
$\Delta W = \Delta K + \Delta U$

$W = F_1 \, \Delta x_1 = p_1 A_1 \, \Delta x_1 = p_1 V$

$K = \frac{1}{2} m v_1^2 = \frac{1}{2} \rho V v_1^2$

$U = m g y_1 = \rho V g y_1$

$p_1 V = - \frac{1}{2} \rho V v_1^2 - \rho V g y_1$



Rearranging

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = constant$$

Applies only to an ideal fluid (zero viscosity)

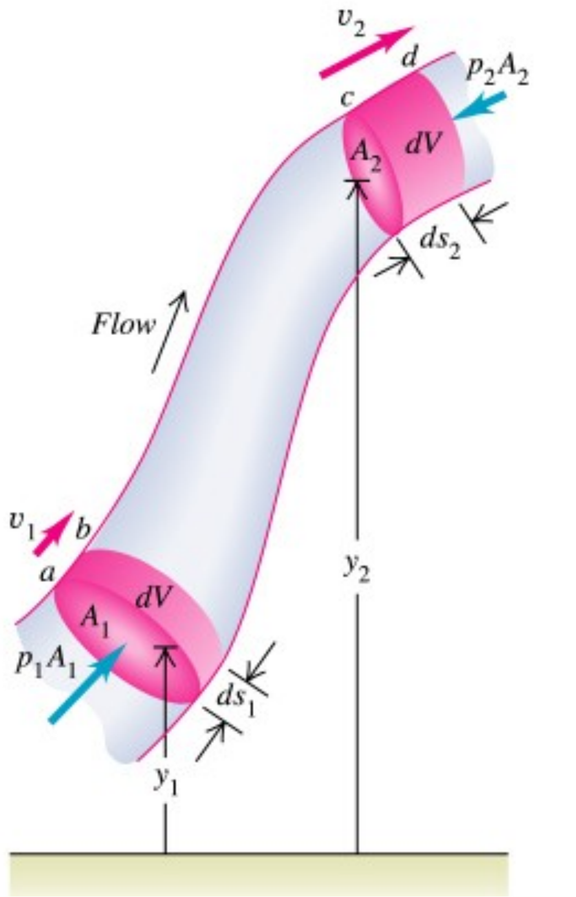
Derivation of Bernoulli's equation

Mass element m moves from (1) to (2)

$m = \rho A_1 \Delta x_1 = \rho A_2 \Delta x_2 = \rho \Delta V$ where $\Delta V = A_1 \Delta x_1 - A_2 \Delta x_2$

Equation of continuity $A V$ = constant

$A_1 v_1 = A_2 v_2 \quad A_1 > A_2 \Rightarrow v_1 < v_2$



Since $v_1 < v_2$ the mass element has been accelerated by the net force

$F_1 - F_2 = p_1 A_1 - p_2 A_2$

Conservation of energy

A pressurized fluid must contain energy by the virtue that work must be done to establish the pressure.

A fluid that undergoes a pressure change undergoes an energy change.

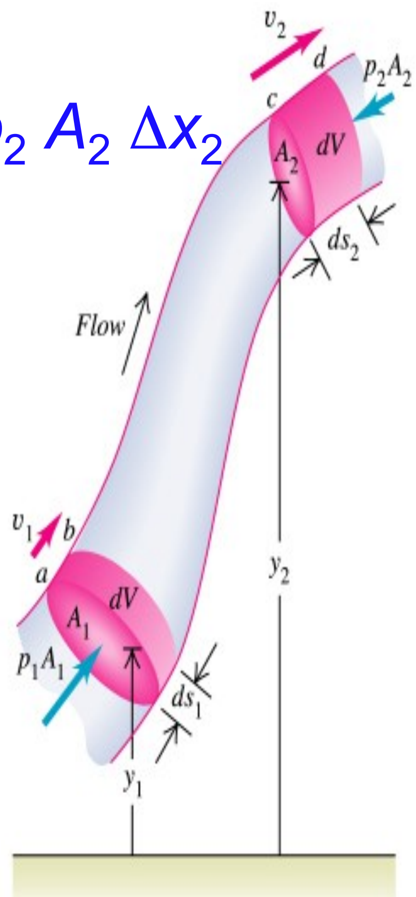
Change in kinetic energy

$$\Delta K = \frac{1}{2} m\, v_2^2 - \frac{1}{2} m\, v_1^2 = \frac{1}{2} \rho\, \Delta V\, v_2^2 - \frac{1}{2} \rho\, \Delta V\, v_1^2$$

Change in potential energy

$$\Delta U = m\, g\, y_2 - m\, g\, y_1 = \rho\, \Delta V\, g\, y_2 - \rho\, \Delta V\, g\, y_1$$

Work done $W_{net} = F_1\, \Delta x_1 - F_2\, \Delta x_2 = p_1\, A_1\, \Delta x_1 - p_2\, A_2\, \Delta x_2$



Work done = change in kinetic + potential energy

$$W_{net} = p_1\, \Delta V - p_2\, \Delta V = \Delta K + \Delta U$$

$$p_1\, \cancel{\Delta V} - p_2\, \cancel{\Delta V} =$$

$$\frac{1}{2} \rho\, \cancel{\Delta V}\, v_2^2 - \frac{1}{2} \rho\, \cancel{\Delta V}\, v_1^2 + \rho\, \cancel{\Delta V}\, g\, y_2 - \rho\, \cancel{\Delta V}\, g\, y_1$$

Rearranging

$$p_1 + \frac{1}{2} \rho\, v_1^2 + \rho\, g\, y_1 = p_2 + \frac{1}{2} \rho\, v_2^2 + \rho\, g\, y_2$$

Applies only to an ideal fluid (zero viscosity)

Bernoulli's Equation

for any point along a flow tube or streamline

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Between any two points along a flow tube or streamline

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Dimensions

p [Pa] = [N.m^{-2}] = [N.m.m^{-3}] = [J.m^{-3}]

$\frac{1}{2}\rho v^2$ [kg.m^{-3}.m^2.s^{-2}] = [kg.m^{-1}.s^{-2}] = [N.m.m^{-3}] = [J.m^{-3}]

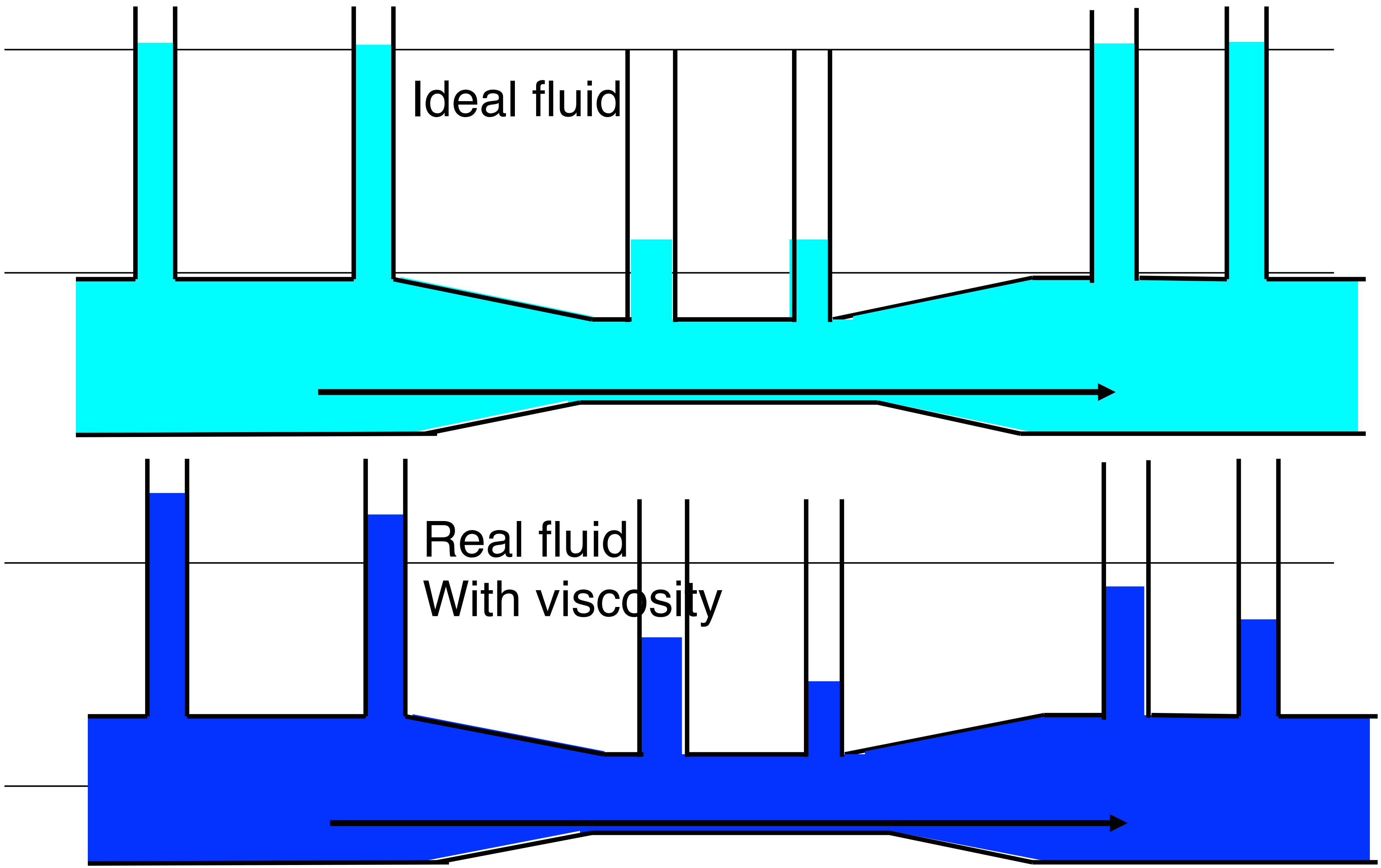
ρ g h [kg.m^{-3} m.s^{-2}. m] = [kg.m.s^{-2}.m.m^{-3}] = [N.m.m^{-3}] = [J.m^{-3}]

Each term has the dimensions of energy / volume or energy density.

$\frac{1}{2}\rho v^2$ KE of bulk motion of fluid

$\rho g h$ GPE for location of fluid

p pressure energy density arising from internal forces within moving fluid (similar to energy stored in a spring)



•In a real fluid the pressure decreases along the pipe.
•Viscous fluids have frictional forces which dissipate energy through heating.