

## Axial Members

- Members with length significantly greater than the largest cross-sectional dimension and with loads applied along the longitudinal axis.

Cables of Mackinaw bridge



Hydraulic cylinders in a dump truck



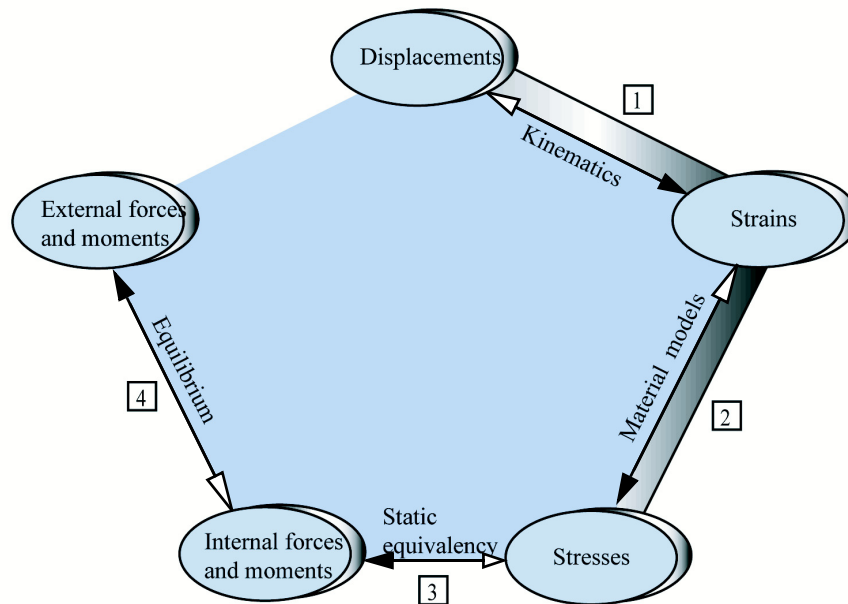
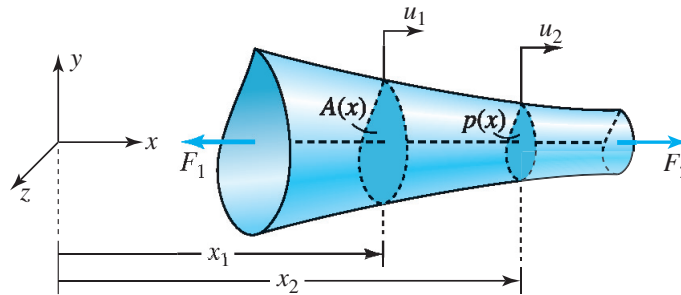
### Learning objectives are:

- Understand the theory, its limitations, and its applications for design and analysis of axial members.
- Develop the discipline to draw free body diagrams and approximate deformed shapes in the design and analysis of structures.

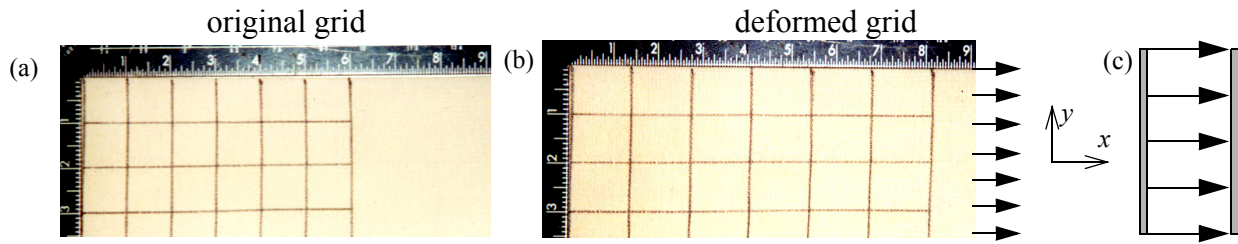
# Theory

## Theory Objective

- to obtain a formula for the relative displacements  $(u_2 - u_1)$  in terms of the internal axial force  $N$ .
- to obtain a formula for the axial stress  $\sigma_{xx}$  in terms of the internal axial force  $N$ .



### Kinematics



Assumption 1 Plane sections remain plane and parallel.  $u = u(x)$

- The displacement  $u$  is considered positive in the positive  $x$ -direction.

Assumption 2 Strains are small.  $\epsilon_{xx} = \frac{du(x)}{dx}$

### Material Model

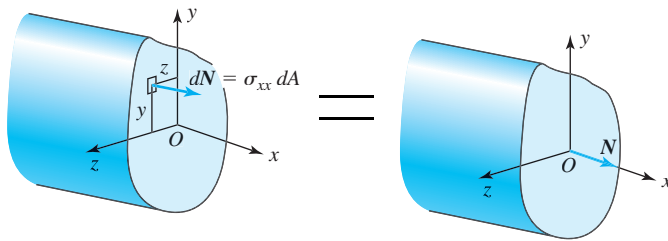
Assumption 3 Material is isotropic.

Assumption 4 Material is linearly elastic.

Assumption 5 There are no inelastic strains.

From Hooke's Law:  $\sigma_{xx} = E\epsilon_{xx}$ , we obtain  $\sigma_{xx} = E\frac{du}{dx}$

### Internal Axial Force



$$N = \int_A \sigma_{xx} dA$$

$$N = \int_A E \frac{du}{dx} dA = \frac{du}{dx} \int_A E dA$$

- For pure axial problems the internal moments (bending)  $M_y$  and  $M_z$  must be zero.
- For homogenous materials all external and internal axial forces must pass through the centroids of the cross-section and all centroids must lie on a straight line.

## Axial Formulas

Assumption 6 Material is homogenous across the cross-section.

$$N = E \frac{du}{dx} \int_A dA = EA \frac{du}{dx} \quad \text{or} \quad \frac{du}{dx} = \frac{N}{EA}$$

$$\sigma_{xx} = E \frac{du}{dx} = E \left( \frac{N}{EA} \right) \quad \text{or} \quad \sigma_{xx} = \frac{N}{A}$$

- The quantity  $EA$  is called the Axial rigidity.

Assumption 7 Material is homogenous between  $x_1$  and  $x_2$ .

Assumption 8 The bar is not tapered between  $x_1$  and  $x_2$ .

Assumption 9 The external (hence internal) axial force does not change with  $x$  between  $x_1$  and  $x_2$ .

$$u_2 - u_1 = \frac{N(x_2 - x_1)}{EA}$$

Two options for determining internal axial force  $N$

- $N$  is always drawn in tension at the imaginary cut on the free body diagram.
  - Positive value of  $\sigma_{xx}$  will be tension.
  - Positive  $u_2 - u_1$  is extension.
  - Positive  $u$  is in the positive  $x$ -direction.
- $N$  is drawn at the imaginary cut in a direction to equilibrate the external forces on the free body diagram.
  - Tension or compression for  $\sigma_{xx}$  has to be determined by inspection.
  - Extension or contraction for  $\delta = u_2 - u_1$  has to be determined by inspection.
  - Direction of displacement  $u$  has to be determined by inspection.

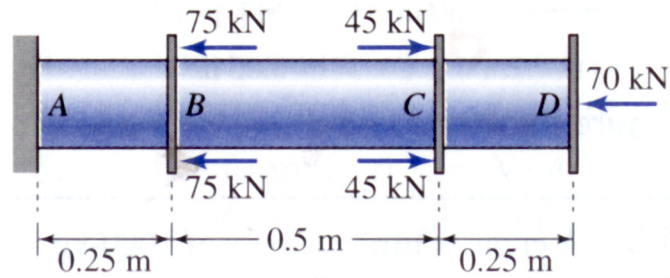
## Axial stresses and strains

- all stress components except  $\sigma_{xx}$  can be assumed zero.

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E}$$

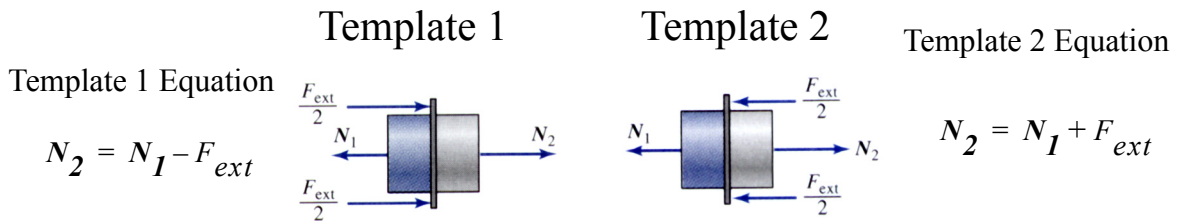
$$\epsilon_{yy} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\epsilon_{xx} \quad \epsilon_{zz} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\epsilon_{xx}$$

**C4.1** Determine the internal axial forces in segments AB, BC, and CD by making imaginary cuts and drawing free body diagrams.

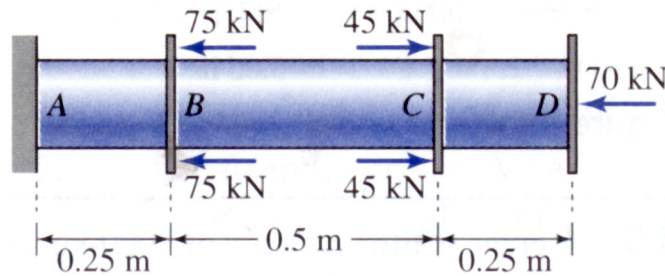


## Axial Force Diagrams

- An axial force diagram is a plot of internal axial force  $N$  vs.  $x$
- Internal axial force jumps by the value of the external force as one crosses the external force from left to right.
- An axial template is used to determine the direction of the jump in  $N$ .
- A template is a free body diagram of a small segment of an axial bar created by making an imaginary cut just before and just after the section where the external force is applied.

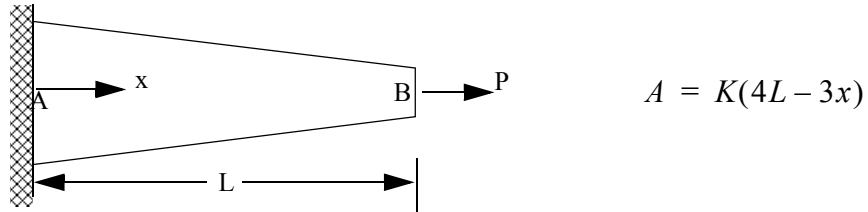


**C4.2** Determine the internal axial forces in segments AB, BC, and CD by drawing axial force diagram.



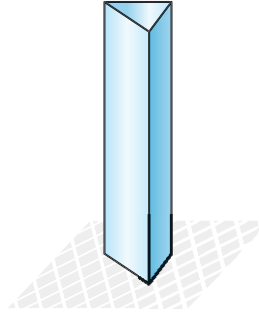
**C4.3** The axial rigidity of the bar in problem 4.8 is  $EA = 80,000 \text{ kN}$ . Determine the movement of section at C.

**C4.4** The tapered bar shown in Fig. C4.4 has a cross-sectional area that varies with  $x$  as given. Determine the elongation of the bar in terms of  $P$ ,  $L$ ,  $E$  and  $K$ .



**Fig. C4.4**

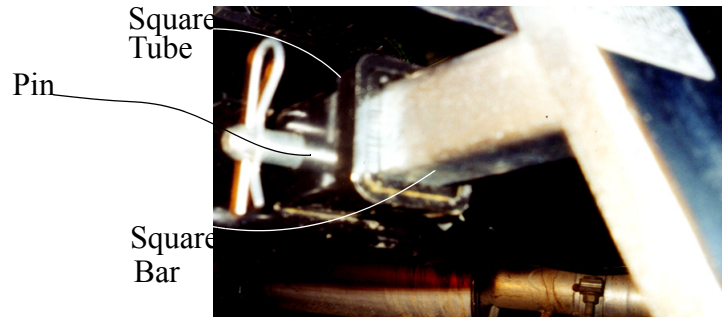
**C4.5** The column shown has a length  $L$ , modulus of elasticity  $E$ , specific weight  $\gamma$ , and length  $a$  as the side of an equilateral triangle. Determine the contraction of the column in terms of  $L$ ,  $E$ ,  $\gamma$ , and  $a$ .



**Fig. C4.5**



**C4.6** A hitch for an automobile is to be designed for pulling a maximum load of 3,600 lbs. A solid-square-bar fits into a square-tube, and is held in place by a pin as shown. The allowable axial stress in the bar is 6 ksi, the allowable shear stress in the pin is 10 ksi, and the allowable axial stress in the steel tube is 12 ksi. To the nearest 1/16th of an inch, determine the minimum cross-sectional dimensions of the pin, the bar and the tube. Neglect stress concentration. (Note: Pin is in double shear)



**Fig. C4.6**

## Structural analysis

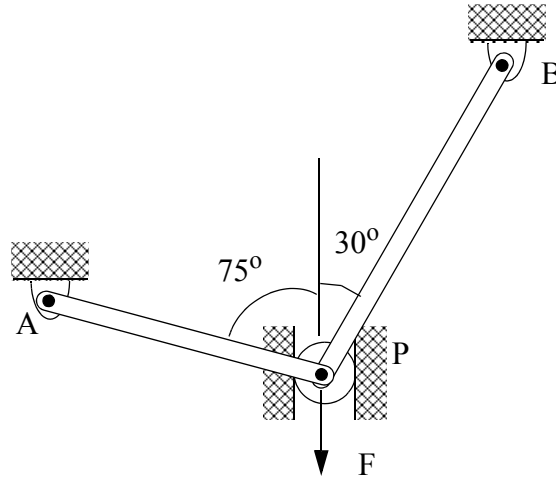
$$\delta = \frac{NL}{EA}$$

- $\delta$  is the deformation of the bar in the **undeformed direction**.
- If  $N$  is a **tensile** force then  $\delta$  is **elongation**.
- If  $N$  is a **compressive** force then  $\delta$  is **contraction**.
- Deformation of a member shown in the drawing of approximate deformed geometry **must be consistent** with the internal force in the member that is shown on the free body diagram.
- In statically indeterminate structures number of unknowns exceed the number of static equilibrium equations. The extra equations needed to solve the problem are relationships between deformations obtained from the deformed geometry.
- **Force method**----Internal forces or reaction forces are unknowns.
- **Displacement method**---Displacements of points are unknowns.

### General Procedure for analysis of indeterminate structures.

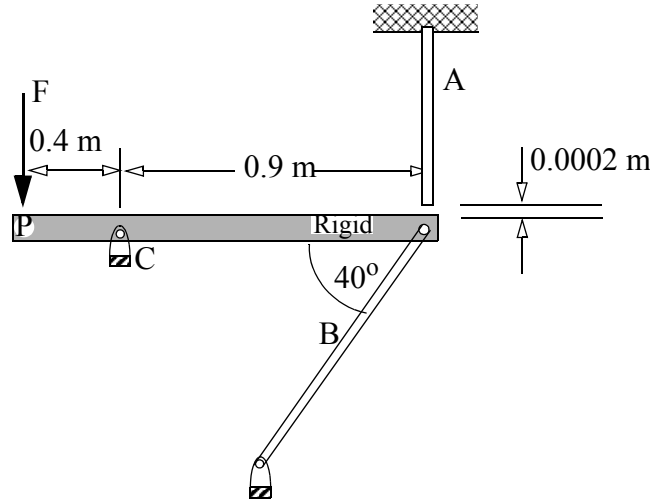
- If there is a gap, assume it will close at equilibrium.
- Draw Free Body Diagrams, write equilibrium equations.
- Draw an exaggerated approximate deformed shape. Write compatibility equations.
- Write internal forces in terms of deformations for each member.
- Solve equations.
- Check if the assumption of gap closure is correct.

**C4.7** A force  $F = 20 \text{ kN}$  is applied to the roller that slides inside a slot. Both bars have an area of cross-section of  $A = 100 \text{ mm}^2$  and a Modulus of Elasticity  $E = 200 \text{ GPa}$ . Bar AP and BP have lengths of  $L_{AP} = 200 \text{ mm}$  and  $L_{BP} = 250 \text{ mm}$  respectively. Determine the displacement of the roller and axial stress in bar A.



**Fig. C4.7**

**C4.8** In Fig. C4.8, a gap exists between the rigid bar and rod A before the force  $F=75\text{ kN}$  is applied. The rigid bar is hinged at point C. The lengths of bar A and B are 1 m and 1.5 m respectively and the diameters are 50 mm and 30 mm respectively. The bars are made of steel with a modulus of elasticity  $E = 200\text{ GPa}$  and Poisson's ratio is 0.28. Determine (a) the deformation of the two bars. (b) the change in the diameters of the two bars.



**Fig. C4.8**

### Class Problem 4.1

Write equilibrium equations, compatibility equations, and  $\delta = \frac{NL}{EA}$  for each member using the given data. **No need to solve.**

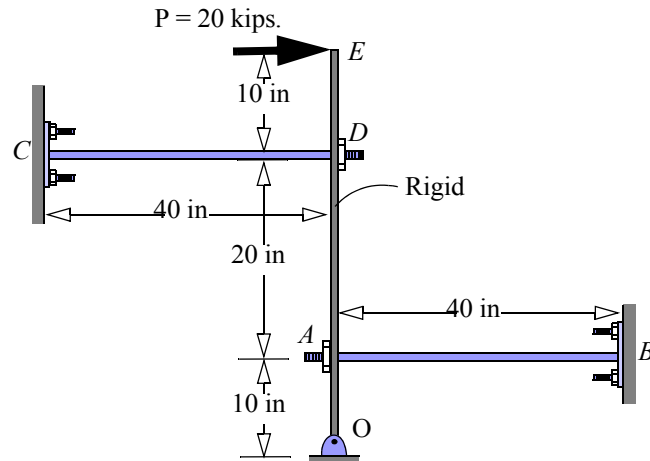
Use displacement of point E  $\delta_E$  as unknown.

$$E = 10,000 \text{ ksi}$$

$$A = 5 \text{ in}^2$$

$$EA = 50,000$$

$$L/EA = 0.8(10^{-3})$$



## Class Problem 4.2

Write equilibrium equations, compatibility equations, and  $\delta = \frac{NL}{EA}$  for each member using the given data. **No need to solve.**

Use reaction force at  $A$  ( $R_A$ ) as unknown.

$$E = 10,000 \text{ ksi} \quad A = 5 \text{ in}^2.$$

$$EA = 50,000$$

